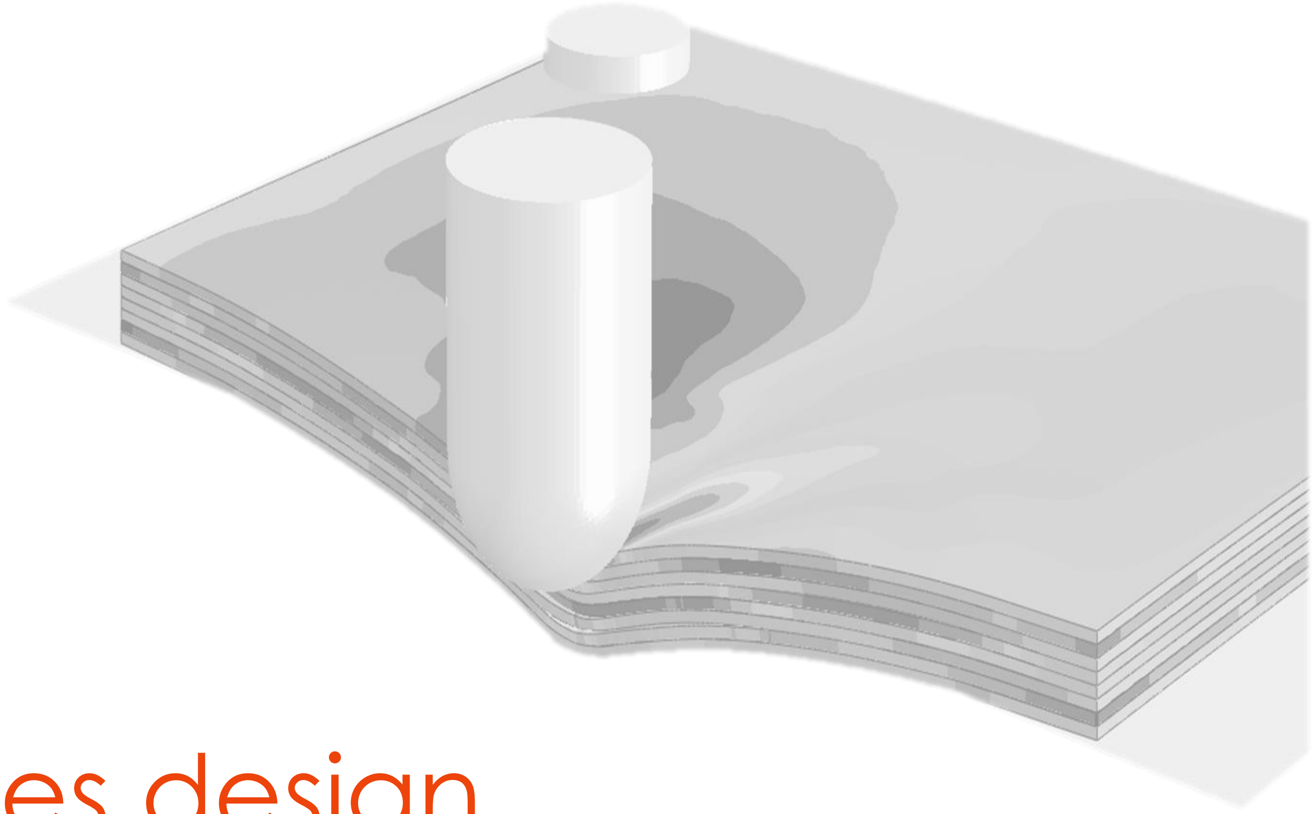


Quasi-Trivial solutions for composite laminates design

T. Garulli

Summer AMADE Day, Girona, 14th July 2023

Quasi-Trivial solutions for laminates design: contents

- Introduction
- Elastic design of laminates
- Quasi-trivial solutions
- Concluding remarks

ANALYSIS AND ADVANCED MATERIALS
FOR STRUCTURAL DESIGN

Introduction

slido

Do you know what a quasi-trivial solution is?

ⓘ Start presenting to display the poll results on this slide.

Do you know what a quasi-trivial solution is?

Multiple Choice Poll · 36 votes · 36 participants

No - 26 votes — 72%

Maybe... - 8 votes — 22%

Of course I do! - 2 votes — 6%

slido

Quasi-Trivial (QT) solutions: discovery & definition

ELSEVIER

Composites Science and Technology 61 (2001) 1465–1473

COMPOSITES SCIENCE AND TECHNOLOGY

www.elsevier.com/locate/compscitech

A special class of uncoupled and quasi-homogeneous laminates

P. Vannucci*, G. Verchery

LRMA, Laboratoire de Recherche en Mécanique et Acoustique, I.S.A.T., Institut Supérieur de l'Automobile et des Transports, Université de Bourgogne, 49, Rue Mademoiselle Bourgeois, BP 31, 58027 Nevers Cedex, France

Received 3 July 2000; received in revised form 20 February 2001; accepted 13 March 2001

Discovered in 2001 by Vannucci and Verchery

"*[…]* ***a particular class of solutions for*** *the two inverse problems of* ***finding uncoupled or quasi-homogeneous laminates*** *made by identical anisotropic layers. These solutions, called* ***quasi-trivial*** *[…]*"

AMADE

ANALYSIS AND ADVANCED MATERIALS
FOR STRUCTURAL DESIGN

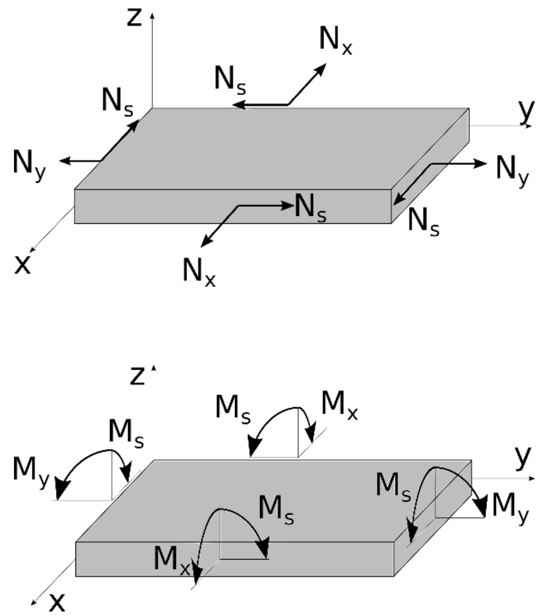
Elastic design of laminates

Laminate constitutive behavior (CLPT)

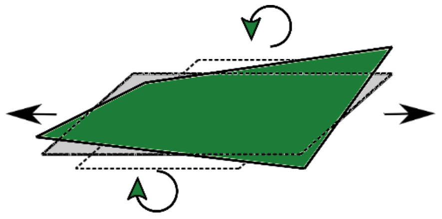
In-plane force resultants; Extensional stiffness matrix; Bending-extension coupling; Mid-plane strains

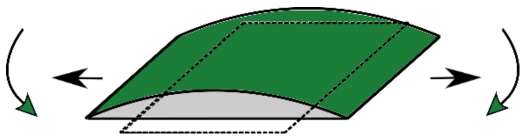
$$\begin{Bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_s^0 \\ \chi_x \\ \chi_y \\ \chi_s \end{Bmatrix}$$

Moments; Bending-extension coupling; Bending stiffness matrix; Curvatures

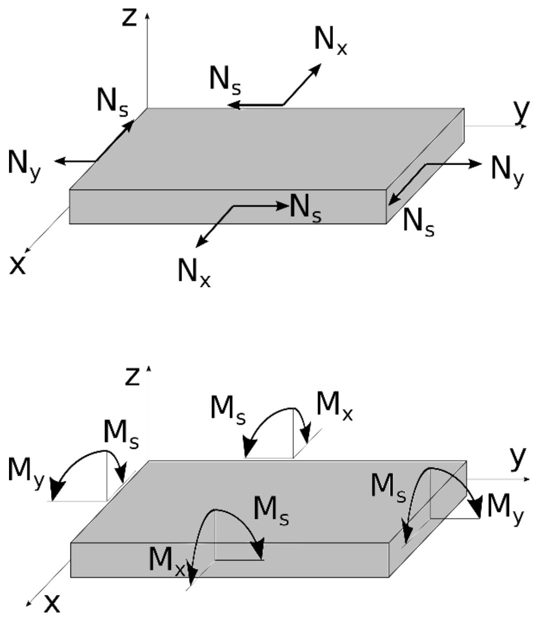


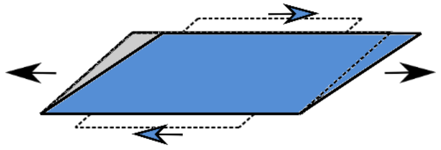
Laminate constitutive behavior (CLPT): the bad guys

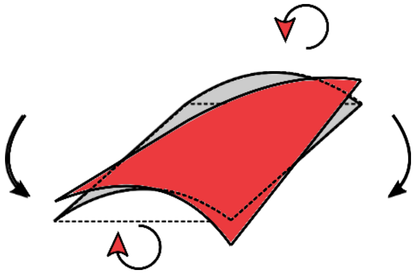




$$\begin{Bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_s^0 \\ \chi_x \\ \chi_y \\ \chi_s \end{Bmatrix}$$







Desirable laminate elastic behaviors

Uncoupling

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Membrane orthotropy

$$\boldsymbol{A} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$$

Bending orthotropy

$$\boldsymbol{D} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}$$

Membrane isotropy

$$\boldsymbol{A} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & \frac{A_{11} - A_{12}}{2} \end{bmatrix}$$

Bending isotropy

$$\boldsymbol{D} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & \frac{D_{11} - D_{12}}{2} \end{bmatrix}$$

slido

Would you be able to design a laminate with ...

ⓘ Start presenting to display the poll results on this slide.

Would you be able to design a laminate with ...

Multiple Choice Poll 25 votes 25 participants

B=0 (uncoupled) - 19 votes — 76%

A orthotropic, B=0 - 15 votes — 60%

A isotropic (quasi-isotropic), B=0 - 18 votes — 72%

D orthotropic, B=0 - 7 votes — 28%

D isotropic, B=0 - 3 votes — 12%

A & D orthotropic, B=0 - 7 votes — 28%

A & D isotropic, B=0 - 2 votes — 8%

slido

Desired properties… … and … … design strategy

Desired properties		Design strategy
$\boldsymbol{B}=\boldsymbol{0}$ (Uncoupled)		Symmetric
$\boldsymbol{A}$ orthotropic		Balanced
$\boldsymbol{A}$ orthotropic, $\boldsymbol{B}=\boldsymbol{0}$		Balanced + symmetric
$\boldsymbol{A}$ isotropic (quasi-isotropic)		Werren & Norris
$\boldsymbol{A}$ isotropic, $\boldsymbol{B}=\boldsymbol{0}$	[60/0/-60], [0/90/±45], … ←	Werren & Norris + symmetric
$\boldsymbol{D}$ orthotropic		Antisymmetric
$\boldsymbol{D}$ orthotropic, $\boldsymbol{B}=\boldsymbol{0}$		?
$\boldsymbol{D}$ isotropic		?
$\boldsymbol{D}$ isotropic, $\boldsymbol{B}=\boldsymbol{0}$		?
$\boldsymbol{A}$ & $\boldsymbol{D}$ orthotropic		Antisymmetric
$\boldsymbol{A}$ & $\boldsymbol{D}$ orthotropic, $\boldsymbol{B}=\boldsymbol{0}$		?
$\boldsymbol{A}$ & $\boldsymbol{D}$ isotropic		?
$\boldsymbol{A}$ & $\boldsymbol{D}$ isotropic, $\boldsymbol{B}=\boldsymbol{0}$		?

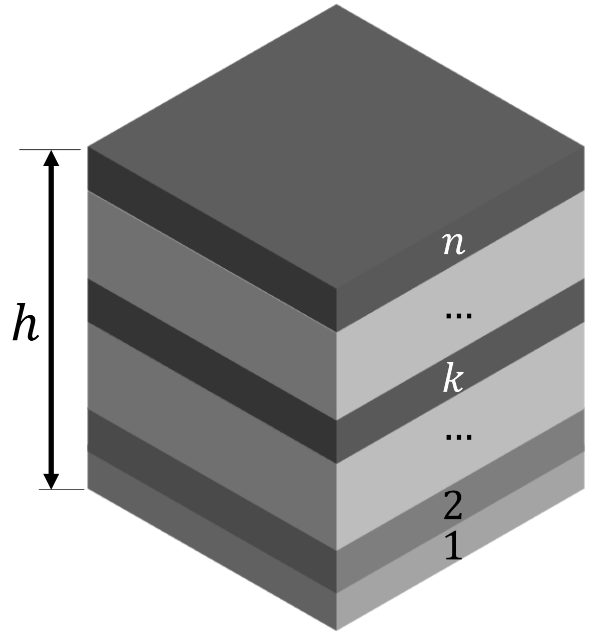
Desired properties… … and … … design strategy

Desired properties		Design strategy
$\boldsymbol{B} = \boldsymbol{0}$ (Uncoupled)		Symmetric
$\boldsymbol{A}$ orthotropic		Balanced
$\boldsymbol{A}$ orthotropic, $\boldsymbol{B} = \boldsymbol{0}$		Balanced + symmetric
$\boldsymbol{A}$ isotropic (quasi-isotropic)		Werren & Norris
$\boldsymbol{A}$ isotropic, $\boldsymbol{B} = \boldsymbol{0}$	[60/0/-60], [0/90/±45], … ←	Werren & Norris + symmetric
$\boldsymbol{D}$ orthotropic		Antisymmetric
$\boldsymbol{D}$ orthotropic, $\boldsymbol{B} = \boldsymbol{0}$		?
$\boldsymbol{D}$ isotropic		?
$\boldsymbol{D}$ isotropic, $\boldsymbol{B} = \boldsymbol{0}$		?
$\boldsymbol{A}$ & $\boldsymbol{D}$ orthotropic		Antisymmetric
$\boldsymbol{A}$ & $\boldsymbol{D}$ orthotropic, $\boldsymbol{B} = \boldsymbol{0}$		?
$\boldsymbol{A}$ & $\boldsymbol{D}$ isotropic		?
$\boldsymbol{A}$ & $\boldsymbol{D}$ isotropic, $\boldsymbol{B} = \boldsymbol{0}$		?

For uncoupling we are restricted to using symmetry

We do not have strategies to control the bending behavior

Stiffness matrices: their definition



$$\boldsymbol{A} = \frac{h}{n}\sum_{k=1}^{n} \boldsymbol{Q}(\delta_k)$$

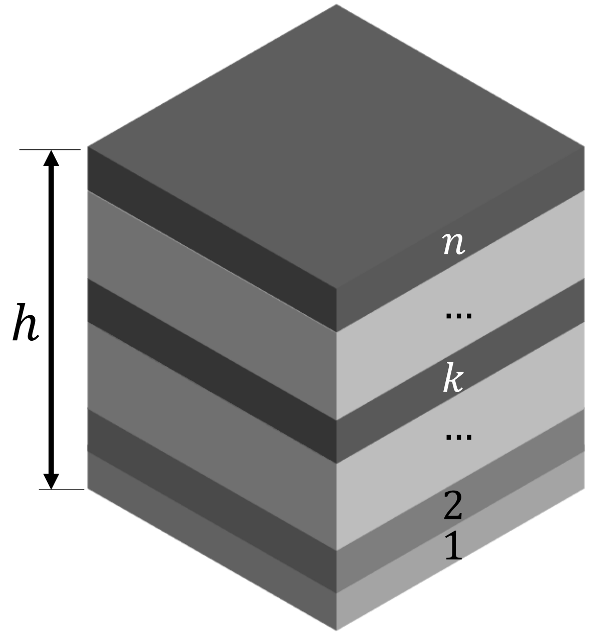
$$\boldsymbol{B} = \frac{1}{2}\frac{h^2}{n^2}\sum_{k=1}^{n} b_k\, \boldsymbol{Q}(\delta_k)$$

$$b_k = 2k - n - 1$$

$$\boldsymbol{D} = \frac{1}{12}\frac{h^3}{n^3}\sum_{k=1}^{n} d_k\, \boldsymbol{Q}(\delta_k)$$

$$d_k = 12k(k - n - 1) + 4 + 3n(n + 2)$$

Stiffness matrices: their definition



$$\boldsymbol{A} = \frac{h}{n}\sum_{k=1}^{n}\boldsymbol{Q}(\delta_k)$$

→ No dependence on ply positions k!

$\boldsymbol{A}$ orthotropic .... Balanced

$\boldsymbol{A}$ isotropic .... W&N layups

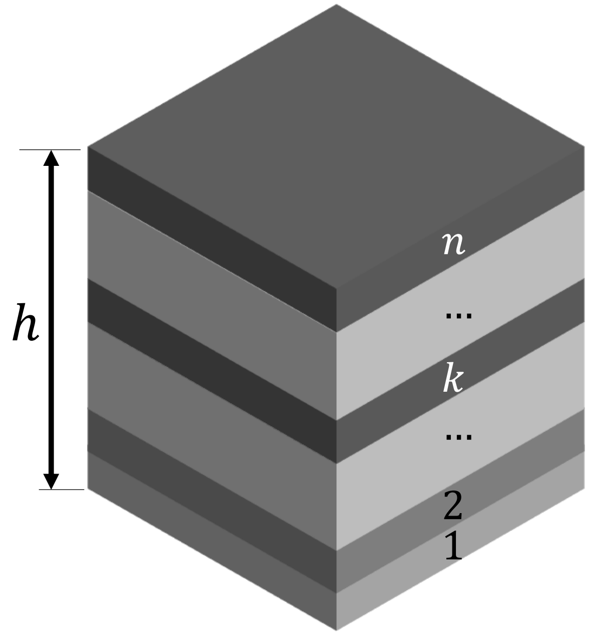
$$\boldsymbol{B} = \frac{1}{2}\frac{h^2}{n^2}\sum_{k=1}^{n} b_k\,\boldsymbol{Q}(\delta_k)$$

$$b_k = 2k - n - 1$$

$$\boldsymbol{D} = \frac{1}{12}\frac{h^3}{n^3}\sum_{k=1}^{n} d_k\,\boldsymbol{Q}(\delta_k)$$

$$d_k = 12k(k - n - 1) + 4 + 3n(n + 2)$$

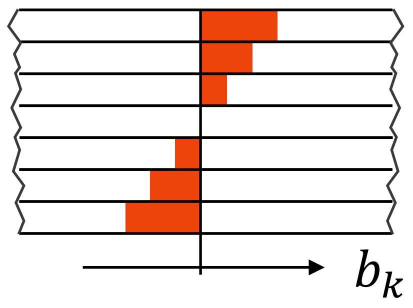
Stiffness matrices: their definition



$$\boldsymbol{A} = \frac{h}{n}\sum_{k=1}^{n} \boldsymbol{Q}(\delta_k)$$

$$\boldsymbol{B} = \frac{1}{2}\frac{h^2}{n^2}\sum_{k=1}^{n} b_k\, \boldsymbol{Q}(\delta_k)$$

$$b_k = 2k - n - 1$$



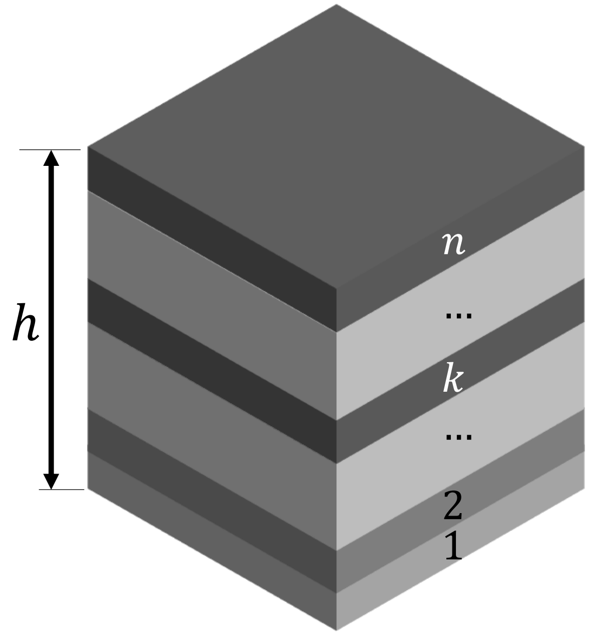
$\boldsymbol{B} = \boldsymbol{0}$

Symmetric

$$\boldsymbol{D} = \frac{1}{12}\frac{h^3}{n^3}\sum_{k=1}^{n} d_k\, \boldsymbol{Q}(\delta_k)$$

$$d_k = 12k(k - n - 1) + 4 + 3n(n + 2)$$

Stiffness matrices: their definition



$$\boldsymbol{A} = \frac{h}{n}\sum_{k=1}^{n} \boldsymbol{Q}(\delta_k)$$

$$\boldsymbol{B} = \frac{1}{2}\frac{h^2}{n^2}\sum_{k=1}^{n} b_k\, \boldsymbol{Q}(\delta_k)$$

$$b_k = 2k - n - 1$$

$$\boldsymbol{D} = \frac{1}{12}\frac{h^3}{n^3}\sum_{k=1}^{n} d_k\, \boldsymbol{Q}(\delta_k)$$

$$d_k = 12k(k - n - 1) + 4 + 3n(n + 2)$$

$\boldsymbol{D}$ orthotropic ······ Antisymmetric

$\boldsymbol{D}$ isotropic ······ ?

AMADE
ANALYSIS AND ADVANCED MATERIALS
FOR STRUCTURAL DESIGN

Quasi-trivial solutions

Uncoupled QT solutions: let's find them!

$\boldsymbol{B} = \frac{1}{2}\frac{h^2}{n^2}\sum_{k=1}^{n} b_k\, \boldsymbol{Q}(\delta_k)$

Sample laminate: $[\vartheta_1/\vartheta_2/\vartheta_2/\vartheta_3/\vartheta_1/\vartheta_1/\vartheta_2]$

Orientations: $\vartheta_1, \vartheta_2, \dots \vartheta_j, \dots, \vartheta_m$

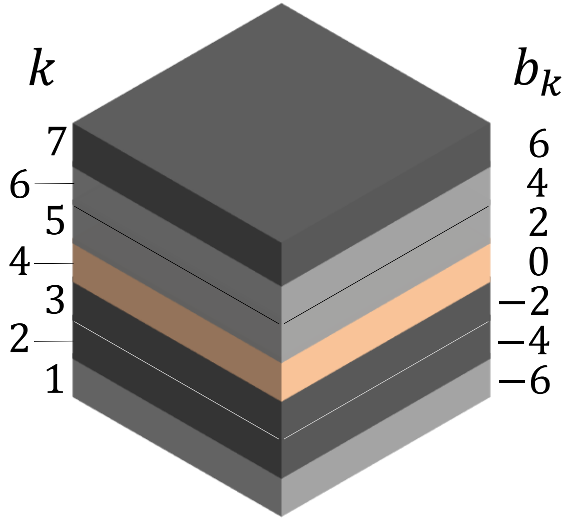
$G_j = \{k: \delta_k = \vartheta_j\}$

Orientations: $\vartheta_1, \vartheta_2, \vartheta_3$

$G_1 = \{1,5,6\}$

$G_2 = \{2,3,7\}$

$G_3 = \{4\}$



$\sum_{j=1}^{m}\sum_{k\in G_j} b_k\, \boldsymbol{Q}(\delta_k) \Rightarrow \sum_{j=1}^{m}\sum_{k\in G_j} b_k\, \boldsymbol{Q}(\vartheta_j)$

$j = 1 \Rightarrow -6 + 2 + 4 = 0$

$\sum_{j=1}^{m} \boldsymbol{Q}(\vartheta_j)\sum_{k\in G_j} b_k$

$j = 2 \Rightarrow -4 - 2 + 6 = 0$

$\sum_{k\in G_j} b_k = 0 \quad \forall\, j = 1, \dots, m \quad \Rightarrow \boldsymbol{B} = \boldsymbol{0}$

$j = 3 \Rightarrow 0$

$\Rightarrow \boldsymbol{B} = \boldsymbol{0}$

QT uncoupled solutions!

Extension-bending-homogeneous and quasi-homogeneous QT solutions

$$\boldsymbol{A} = \frac{h}{n}\sum_{k=1}^{n} \boldsymbol{Q}(\delta_k) \qquad \boldsymbol{A}^* = \frac{\boldsymbol{A}}{h}$$

$$\boldsymbol{B} = \frac{1}{2}\frac{h^2}{n^2}\sum_{k=1}^{n} b_k\, \boldsymbol{Q}(\delta_k)$$

$$\boldsymbol{D} = \frac{1}{12}\frac{h^3}{n^3}\sum_{k=1}^{n} d_k\, \boldsymbol{Q}(\delta_k) \qquad \boldsymbol{D}^* = 12\frac{\boldsymbol{D}}{h^3}$$

$$\boldsymbol{C} = \boldsymbol{A}^* - \boldsymbol{D}^* = \frac{1}{n^3}\sum_{k=1}^{n} c_k \boldsymbol{Q}(\delta_k) \qquad c_k = n^2 - d_k$$

$$\sum_{k\in G_j} c_k = 0 \quad \forall\, j = 1, \dots, m \quad \Rightarrow \boldsymbol{C} = \boldsymbol{0}$$

QT extension-bending homogeneous solutions!

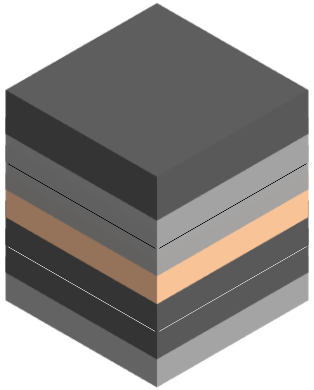
$$\begin{cases} \sum_{k\in G_j} b_k = 0 \quad \forall\, j = 1, \dots, m \quad \Rightarrow \boldsymbol{B} = \boldsymbol{0} \\ \sum_{k\in G_j} c_k = 0 \quad \forall\, j = 1, \dots, m \quad \Rightarrow \boldsymbol{C} = \boldsymbol{0} \end{cases}$$

QT quasi-homogeneous solutions!

QT solutions: a few remarks

- No dependence on $\boldsymbol{Q}(\delta_k)$:
  - $\sum_{k\in G_j} b_k = 0 \quad \forall\, j = 1, \dots, m \quad \Rightarrow \boldsymbol{B} = \boldsymbol{0}$
  - $\sum_{k\in G_j} c_k = 0 \quad \forall\, j = 1, \dots, m \quad \Rightarrow \boldsymbol{C} = \boldsymbol{0}$

    ⇒ orientations can be chosen freely



- $\boldsymbol{C} = \boldsymbol{0} \quad \Rightarrow \quad \boldsymbol{A}^* = \boldsymbol{D}^*$

  ⇒ bending behavior can be designed by together with extension one!

- Symmetric layups are a (small!) sub-class of uncoupled QT solutions

	N. of groups m														
n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
7	0	1	1	0	0	0	0	0	0	0	0	0	0	0	2
8	1	0	1	0	0	0	0	0	0	0	0	0	0	0	2
9	0	1	2	1	0	0	0	0	0	0	0	0	0	0	4
10	0	4	0	1	0	0	0	0	0	0	0	0	0	0	5
11	0	0	9	4	1	0	0	0	0	0	0	0	0	0	14
12	1	8	9	0	1	0	0	0	0	0	0	0	0	0	19
13	0	0	25	32	6	1	0	0	0	0	0	0	0	0	64
14	0	37	34	17	0	1	0	0	0	0	0	0	0	0	89
15	0	0	10	207	78	9	1	0	0	0	0	0	0	0	305
16	0	58	305	96	29	0	1	0	0	0	0	0	0	0	489
17	0	0	2	893	895	144	12	1	0	0	0	0	0	0	1947
18	0	114	1492	1262	208	45	0	1	0	0	0	0	0	0	3122
19	0	0	0	2216	8192	2663	264	16	1	0	0	0	0	0	13352
20	0	0	7391	11240	3683	396	66	0	1	0	0	0	0	0	22777
21	0	0	0	4936	59701	39986	6283	406	20	1	0	0	0	0	111333
22	0	0	29144	101207	49008	8869	694	93	0	1	0	0	0	0	189016
23	0	0	0	6369	346057	519231	141298	13130	626	25	1	0	0	0	1026737
24	0	0	75421	844224	665507	156300	18569	1118	126	0	1	0	0	0	1761266
25	0	0	0	3863	1775560	6116700	2797033	388970	24060	893	30	1	0	0	11107110
26	0	0	96098	6277657	8836070	2900569	410040	35272	1708	166	0	1	0	0	18557581
27	0	0	0	660	6978620	61170759	51236513	10978670	941503	41907	1261	36	1	0	131349930
28	0	0	136700	40159296	112753933	54164504	9788692	940584	62404	2520	214	0	1	0	218008848
29	0	0	0	20	21692599	561464759	868233466	285533218	34157728	1974630	66910	1682	42	1	1773125055

Table 4.1: Number of independent QT uncoupled solutions obtained as a function of the total number of plies, n, and of the number of saturated orientation groups, m. (Symmetric solutions)

Desired properties… … vs … …design strategy

Desired properties	Design strategy
$\boldsymbol{B} = \mathbf{0}$ (Uncoupled)	QT ($\boldsymbol{B} = \mathbf{0}$)
$\boldsymbol{A}$ orthotropic	Balanced
$\boldsymbol{A}$ orthotropic, $\boldsymbol{B} = \mathbf{0}$	Balanced + QT ($\boldsymbol{B} = \mathbf{0}$)
$\boldsymbol{A}$ isotropic (quasi-isotropic)	Werren & Norris
$\boldsymbol{A}$ isotropic, $\boldsymbol{B} = \mathbf{0}$	Werren & Norris + QT ($\boldsymbol{B} = \mathbf{0}$)
$\boldsymbol{D}$ orthotropic	Antisymmetric / balanced + QT ($\mathbf{C} = \mathbf{0}$)
$\boldsymbol{D}$ orthotropic, $\boldsymbol{B} = \mathbf{0}$	Balanced + QT ($\boldsymbol{B} = \mathbf{0}$, $\mathbf{C} = \mathbf{0}$)
$\boldsymbol{D}$ isotropic	Werren & Norris + QT ($\mathbf{C} = \mathbf{0}$)
$\boldsymbol{D}$ isotropic, $\boldsymbol{B} = \mathbf{0}$	Werren & Norris + QT ($\boldsymbol{B} = \mathbf{0}$, $\mathbf{C} = \mathbf{0}$)
$\boldsymbol{A}$ & $\boldsymbol{D}$ orthotropic	Antisymmetric / Balanced + QT ($\mathbf{C} = \mathbf{0}$)
$\boldsymbol{A}$ & $\boldsymbol{D}$ orthotropic, $\boldsymbol{B} = \mathbf{0}$	Balanced + QT ($\boldsymbol{B} = \mathbf{0}$, $\mathbf{C} = \mathbf{0}$)
$\boldsymbol{A}$ & $\boldsymbol{D}$ isotropic	Werren & Norris + QT ($\mathbf{C} = \mathbf{0}$)
$\boldsymbol{A}$ & $\boldsymbol{D}$ isotropic, $\boldsymbol{B} = \mathbf{0}$	Werren & Norris + QT ($\boldsymbol{B} = \mathbf{0}$, $\mathbf{C} = \mathbf{0}$)

Desired properties… … vs … …design strategy

Desired properties	Design strategy
$\boldsymbol{B}=\mathbf{0}$ (Uncoupled)	QT ($\boldsymbol{B}=\mathbf{0}$)
$\boldsymbol{A}$ orthotropic	Balanced
$\boldsymbol{A}$ orthotropic, $\boldsymbol{B}=\mathbf{0}$	Balanced + QT ($\boldsymbol{B}=\mathbf{0}$)
$\boldsymbol{A}$ isotropic (quasi-isotropic)	Werren & Norris
$\boldsymbol{A}$ isotropic, $\boldsymbol{B}=\mathbf{0}$	Werren & Norris + QT ($\boldsymbol{B}=\mathbf{0}$)
$\boldsymbol{D}$ orthotropic	Antisymmetric / balanced + QT ($\mathbf{C}=\mathbf{0}$)
$\boldsymbol{D}$ orthotropic, $\boldsymbol{B}=\mathbf{0}$	Balanced + QT ($\boldsymbol{B}=\mathbf{0}$, $\mathbf{C}=\mathbf{0}$)
$\boldsymbol{D}$ isotropic	Werren & Norris + QT ($\mathbf{C}=\mathbf{0}$)
$\boldsymbol{D}$ isotropic, $\boldsymbol{B}=\mathbf{0}$	Werren & Norris + QT ($\boldsymbol{B}=\mathbf{0}$, $\mathbf{C}=\mathbf{0}$)
$\boldsymbol{A}$ & $\boldsymbol{D}$ orthotropic	Antisymmetric / Balanced + QT ($\mathbf{C}=\mathbf{0}$)
$\boldsymbol{A}$ & $\boldsymbol{D}$ orthotropic, $\boldsymbol{B}=\mathbf{0}$	Balanced + QT ($\boldsymbol{B}=\mathbf{0}$, $\mathbf{C}=\mathbf{0}$)
$\boldsymbol{A}$ & $\boldsymbol{D}$ isotropic	Werren & Norris + QT ($\mathbf{C}=\mathbf{0}$)
$\boldsymbol{A}$ & $\boldsymbol{D}$ isotropic, $\boldsymbol{B}=\mathbf{0}$	Werren & Norris + QT ($\boldsymbol{B}=\mathbf{0}$, $\mathbf{C}=\mathbf{0}$)

Much wider design space for all situations where uncoupling is required

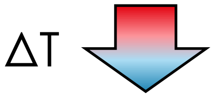
Design strategies for bending behavior become trivial with QT with C=0

AMADE

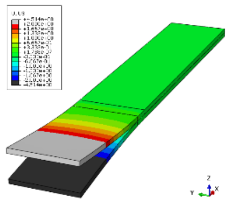
ANALYSIS AND ADVANCED MATERIALS FOR STRUCTURAL DESIGN

Concluding remarks

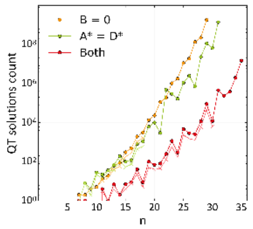
Concluding remarks



- A few things that were not mentioned:
  - Thermo-hygro-elasticity
  - More subtle properties
    - Less relevant to laminates design



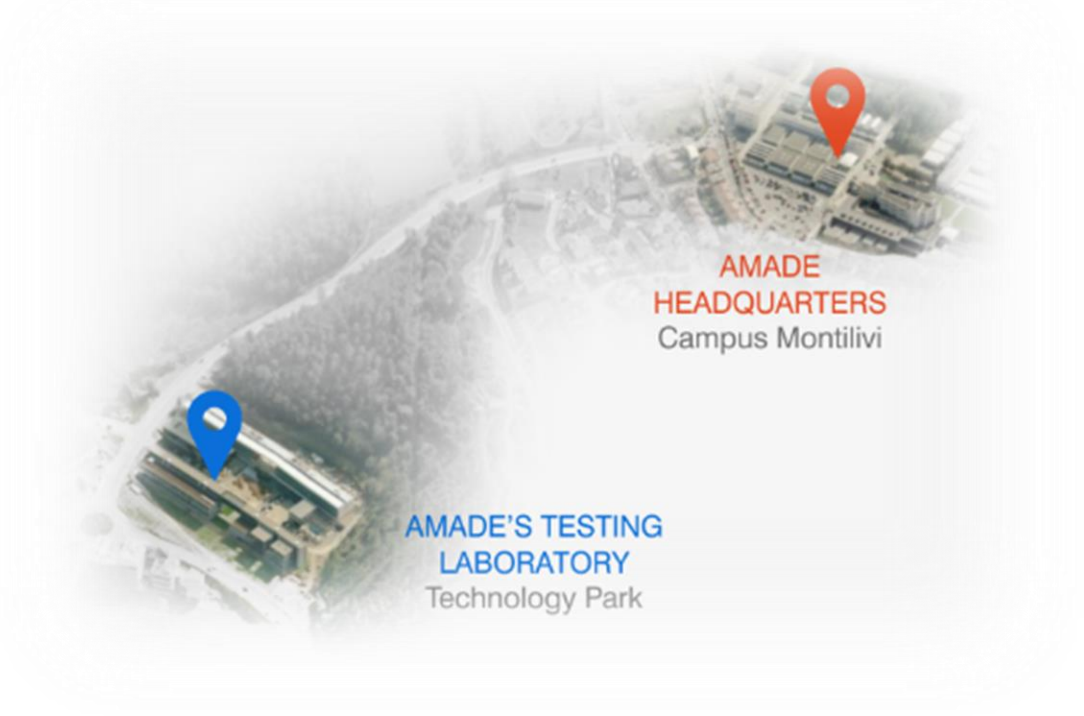
- Successful applications:
  - Fully-Uncoupled Multi-Directional delamination specimens
  - Optimisation problems
  - Design of VAT laminates



- Limitations to adoption:
  - Limited availability
  - Complex search algorithm required
    - Computational power limitations

Funded by
the European Union

Any dissemination of results/communication activity related to the project reflects only the author's view.



Thank you
for your attention!

Universitat de Girona

AMADE
ANALYSIS AND ADVANCED MATERIALS FOR STRUCTURAL DESIGN

U.PORTO
FEUP FACULDADE DE ENGENHARIA UNIVERSIDADE DO PORTO

Accredited Nadcap™
NON-METALLIC MATERIALS TESTING

Part of:
tecnio catalonia
ACCIÓ Generalitat de Catalunya