Definition of Multidimensional Reduced Order Models for the Elastic Analysis of Large Composite Structures

Work made by Francesc Turon for his PhD

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UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

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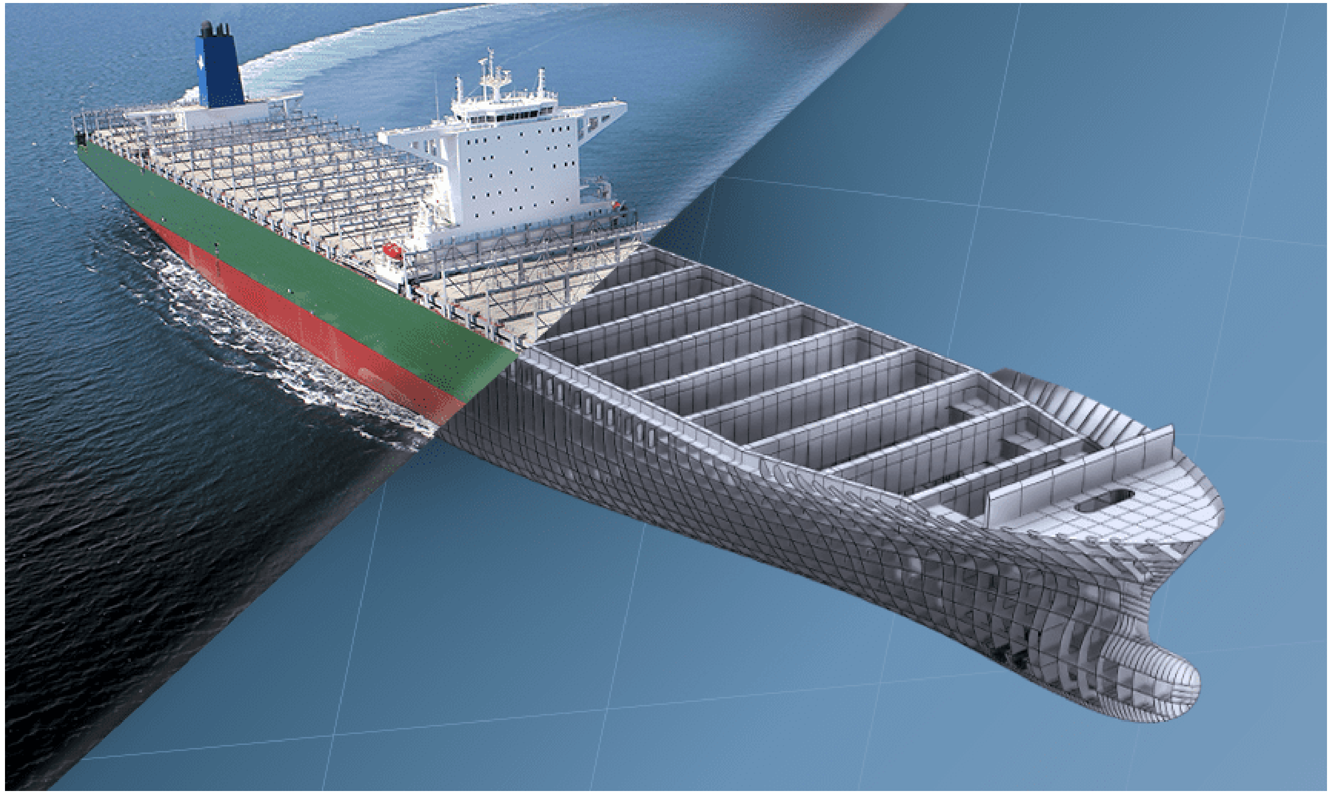
AMADE Days
July 11th 2024

Motivation

There is a need for **efficient computational methods to characterize large composite structures**



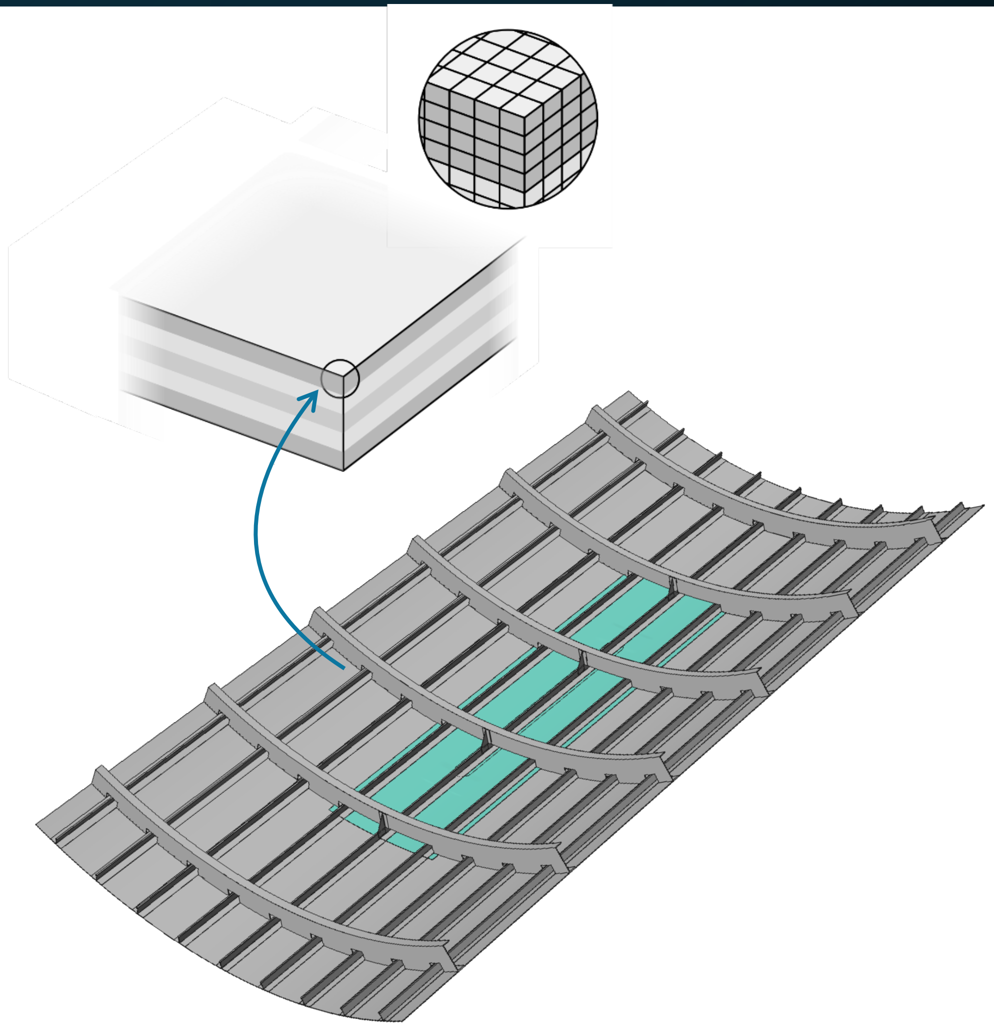
Motivation

The **nature** and **manufacturing procedure** of Large Composite Structures lead to multilayered laminates with shapes that are **complex** to discretize as a **solid model** with volumetric elements.

The elements distribution must **fulfill two** main **requirements**.

- The discretization must be such able to **capture the variations** of the fields involved in the problem.
- Ensure a certain **proportionality** between its **edges**.

The total number of elements and degrees of freedom of the system became **unaffordable** from a **computational cost** point of view even in **elastic regimes**.



Motivation

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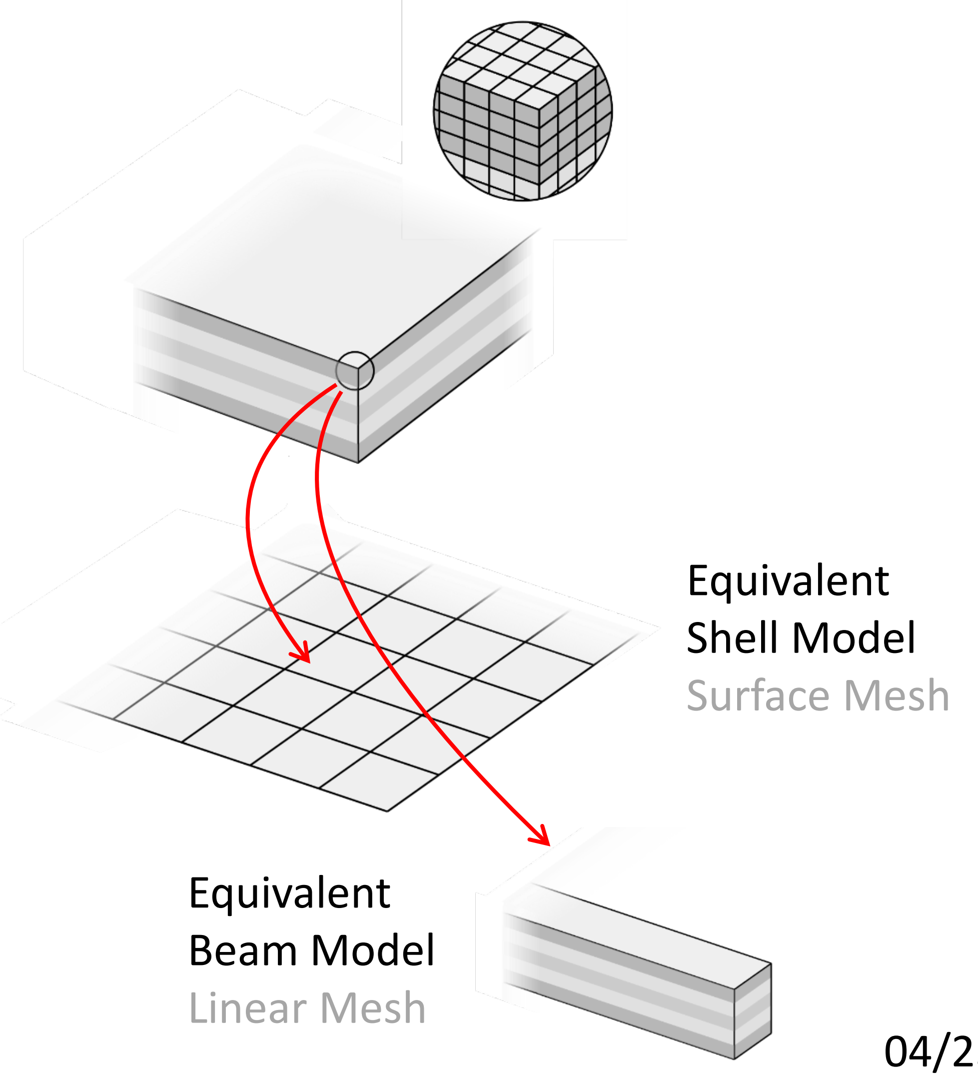
Alternative: Reduced Dimensional Models based on **kinematic theories** to parameterize the real kinematics of the structure and **reduce the number** of **D**egrees **O**f **F**reedom (**DOF**), i.e. Euler and Timoshenko Beam Models and Kirchhoff–Love and Reissner-Mindlin Plate Models.

They **require** certain specific **geometrical features** such as cross-sectioning or laminations with a certain continuity like **regular laminates** or single plies.



Motivation

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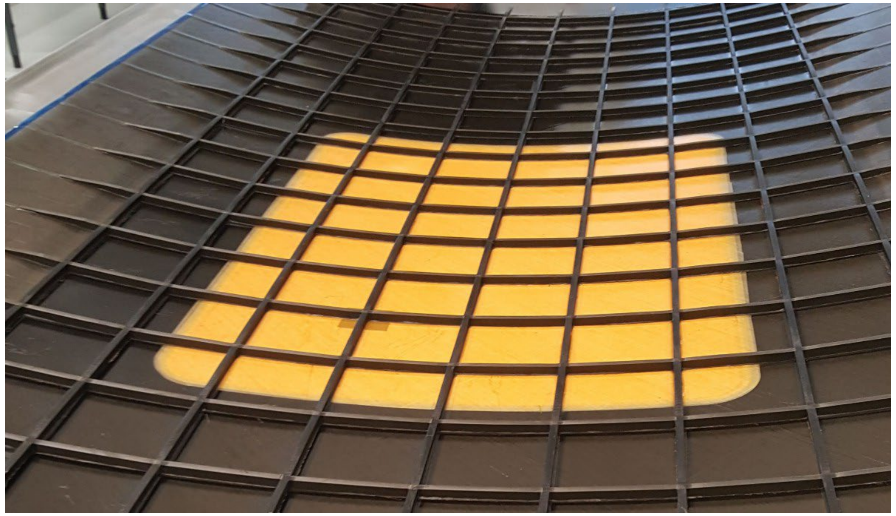
They **require** certain specific **geometrical features** such as cross-sectioning or laminations with a certain continuity like **regular laminates** or single plies.

In the case of **not complying** with the geometrical characteristics it is necessary to rely on the **costly analysis** of the structure **with volumetric mesh**.



Panel made for ACASIAS EU funded project

Motivation

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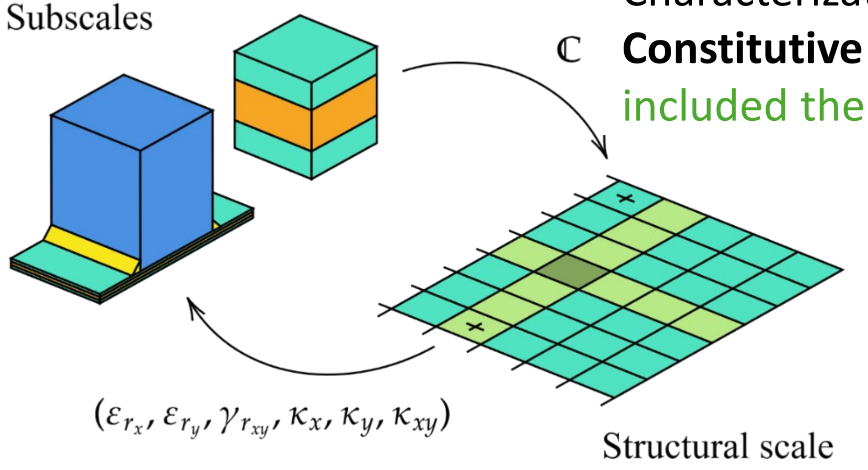
The main objective of this work is to **improve the analysis accuracy** of Large Composite Structures and **reduce the computational cost** related to their simulation.

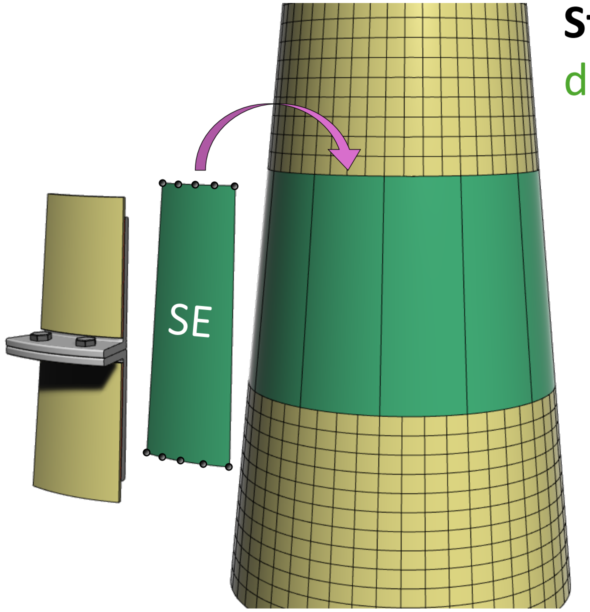
The following to two approaches are proposed:

Characterization at **Constitutive** level included the irregularities



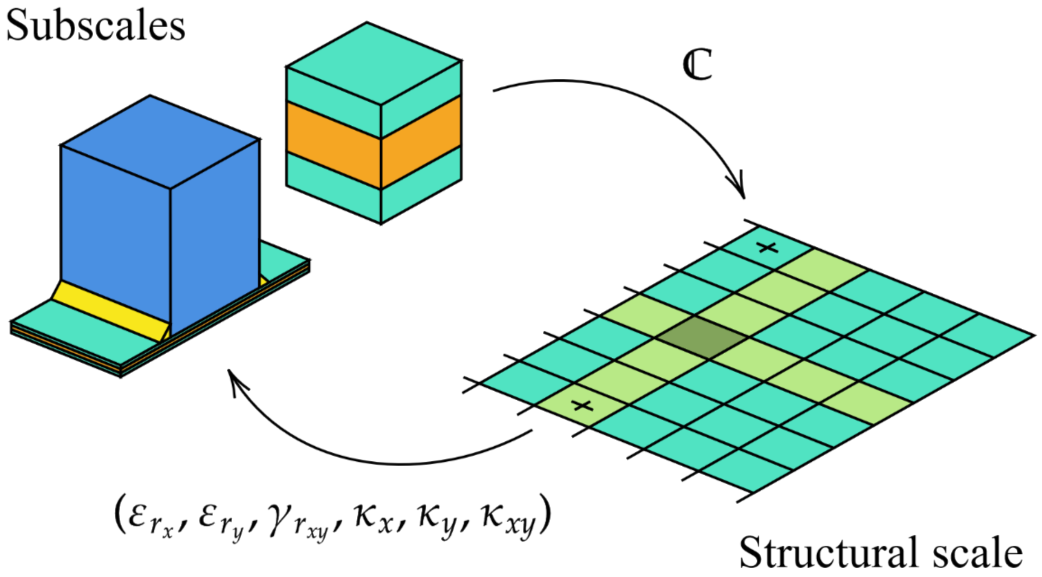
Multi-scale approach

Characterization at **Stiffness** level defining layered ROM



Definition of **ROM** using **M**ixing **D**imensional **C**oupling (MDC)

MULTI-SCALE APPROACH

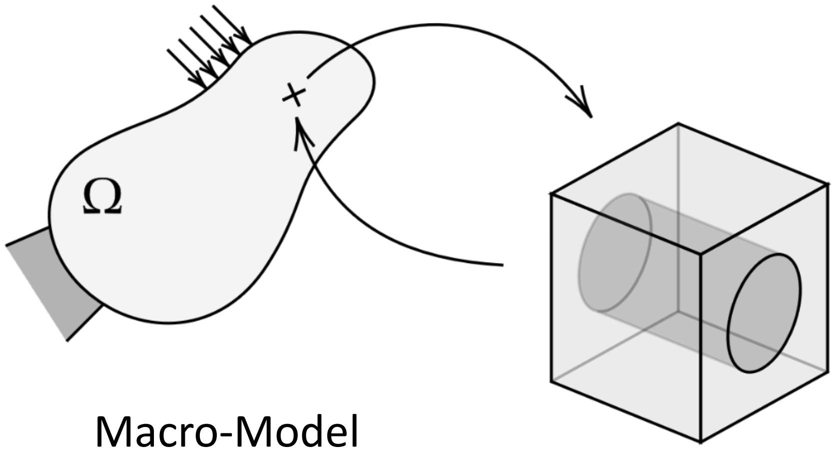


Multi-scale approach

Multi-Scale Approach

The Multi-scale analysis distinguishes **between two scales**, separated by various orders of magnitude in size and are known as micro- and macro- scales.

- The **macro-scale** encloses the entire analyzed structure with a Macro-model.
- The **micro-scale** is meant to be small enough for **capturing** the **physiognomy** of the material used in the structure represented by the macro-scale.



Macro-Model

Micro-Model

Average Strain Theorem:

$$\boldsymbol{\varepsilon}_{\Omega} = \frac{1}{V_{\mu}} \int_{\Omega_{\mu}} \boldsymbol{\varepsilon}_{\mu} \, dV$$

Hill Mandel Theorem:

$$\boldsymbol{\sigma} \cdot \delta\boldsymbol{\varepsilon}_{\Omega} = \frac{1}{V_{\mu}} \int_{\Omega_{\mu}} \boldsymbol{\sigma}_{\mu} \cdot \delta\boldsymbol{\varepsilon}_{\mu} \, dV$$

Boundary **V**alue **P**roblem

Multi-Scale Approach

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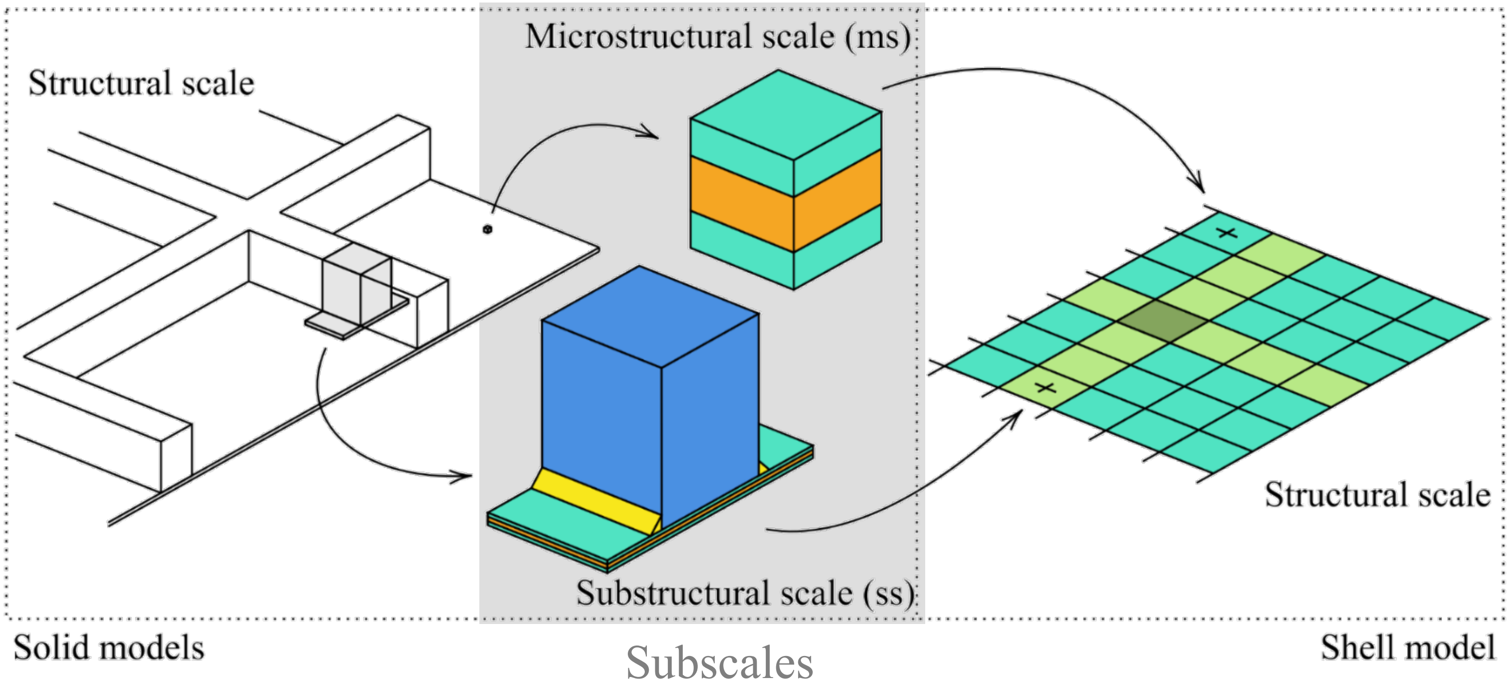
In the proposed approach it is possible to **identify three** different dimensional scales

- The **Structural** represents the structure with a **Shell Model**
- The **Substructural** scale is used to represent the discontinuities in the structure that extends along the laminate.
- The **Microstructural** includes the periodical repetitive patterns in the regular laminate regions.



The former **scale** is defined as a shell model and the latter **subscales** are defined as a solid model.

Multi-Scale Approach

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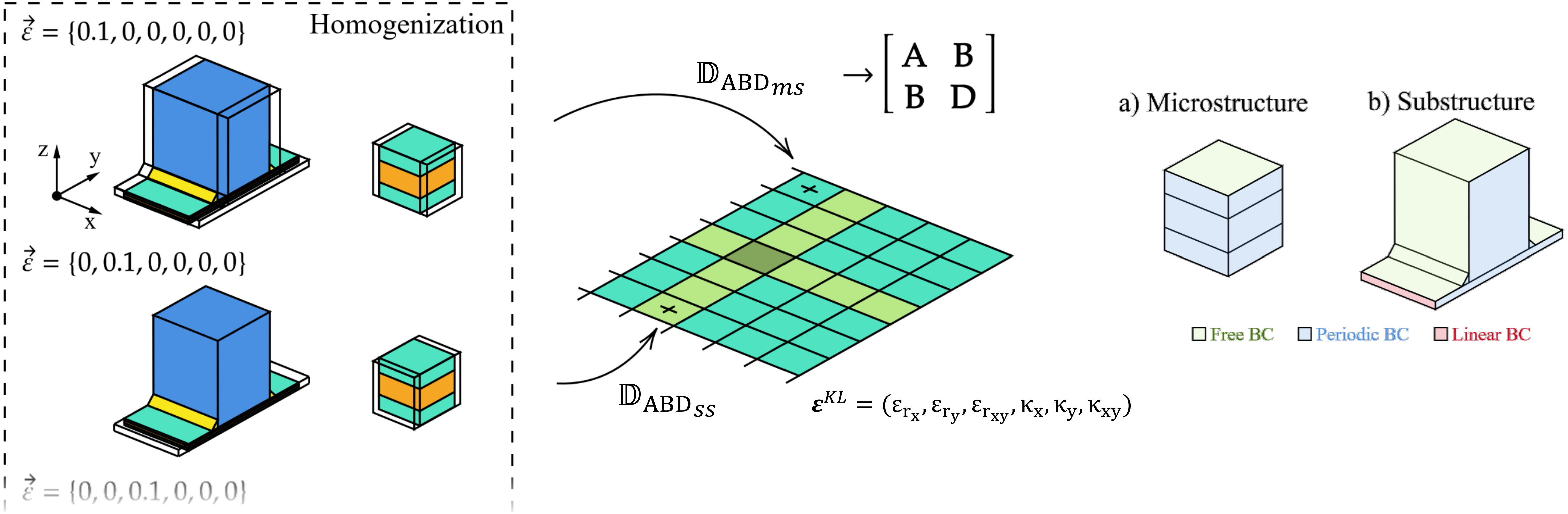
The constitutive behavior of the shell elements, at the structural level, are **characterized** with a **homogenization procedure** that use the substructural and microstructural scales.



Multi-Scale Approach

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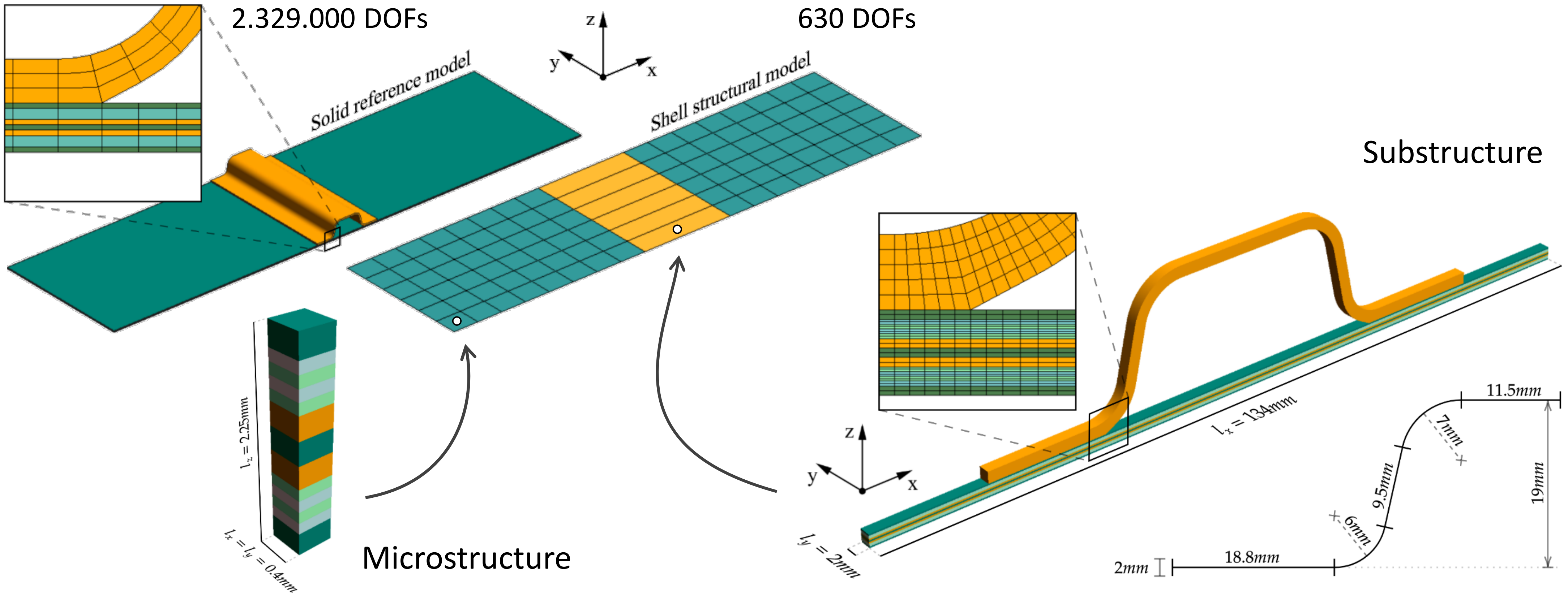
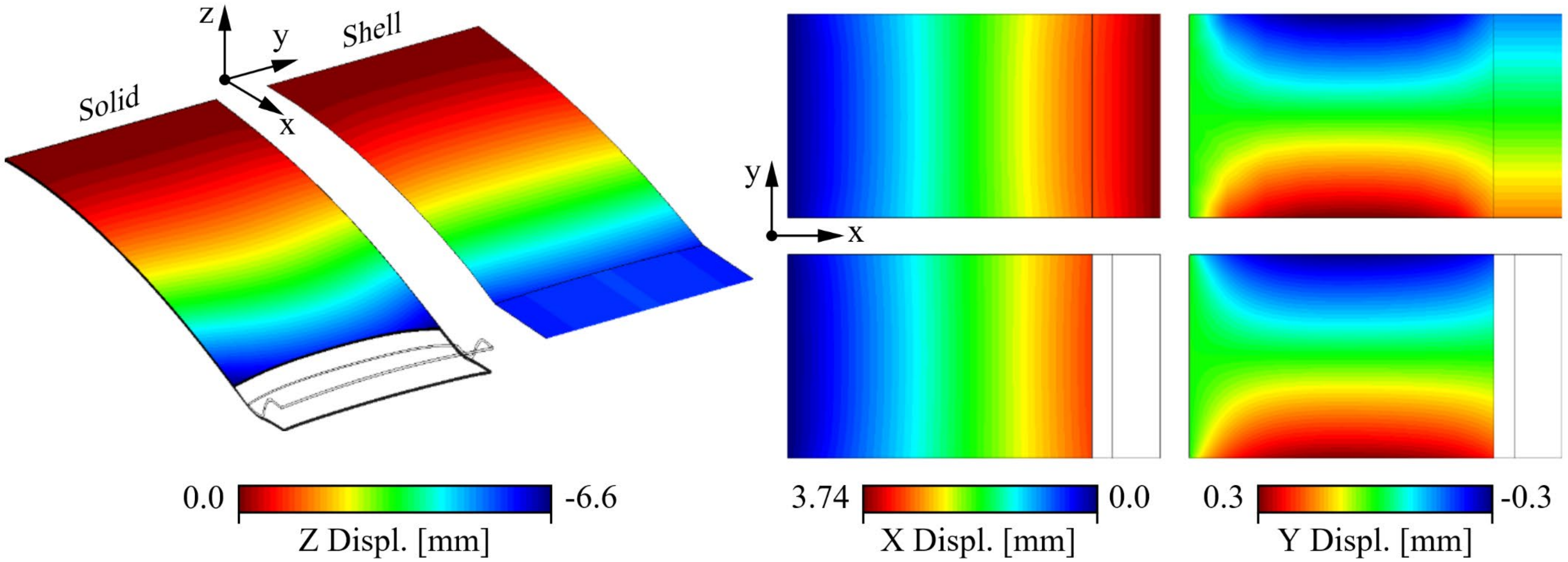
To validate the proposed characterization the following comparison between a full Solid Reference Model and its Multiscale Analysis is performed



Multi-Scale Approach

A **good correlation** can be appreciated between the **displacement field** of both models as the resultant displacement fields only differ on the edges of the laminate next to the transition elements.



Multi-Scale Approach

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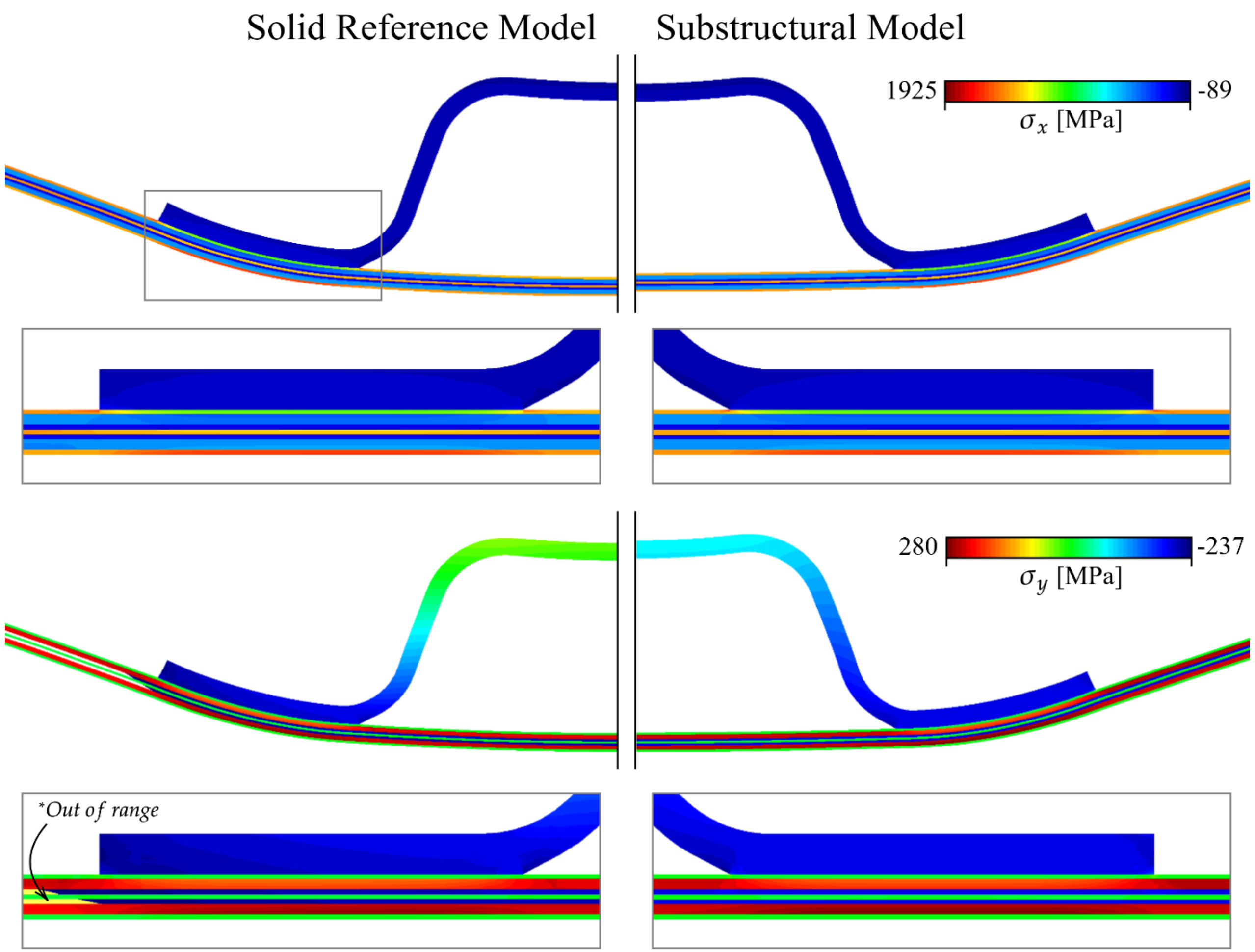
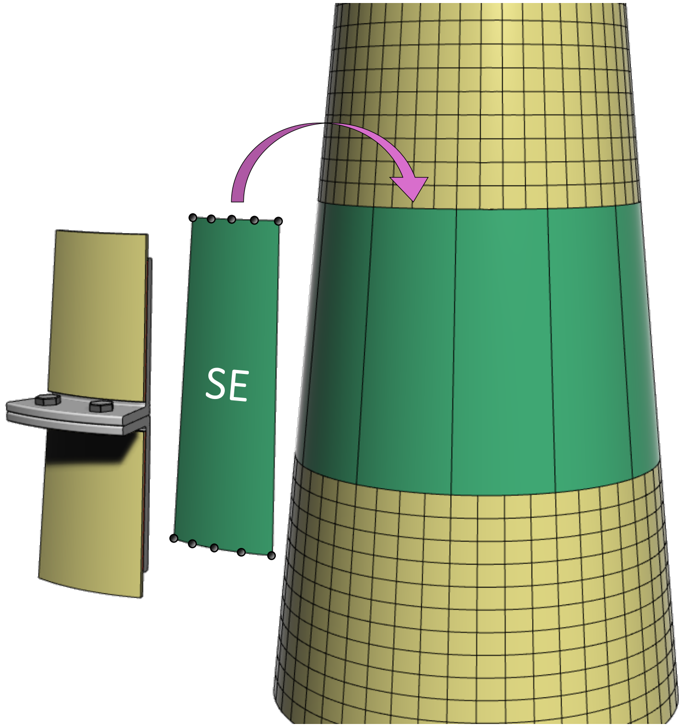


There is a very **good correlation** in the stress values, and its distribution, between both models, layer by layer and in the reinforcement.

As for the stresses obtained in y-direction, they are similar in their distribution, specially in the laminate.

ROM using Mixing Dimensional Coupling



Definition of **ROM** using **M**ixing **D**imensional **C**oupling (MDC)

ROM using Mixing Dimensional Coupling

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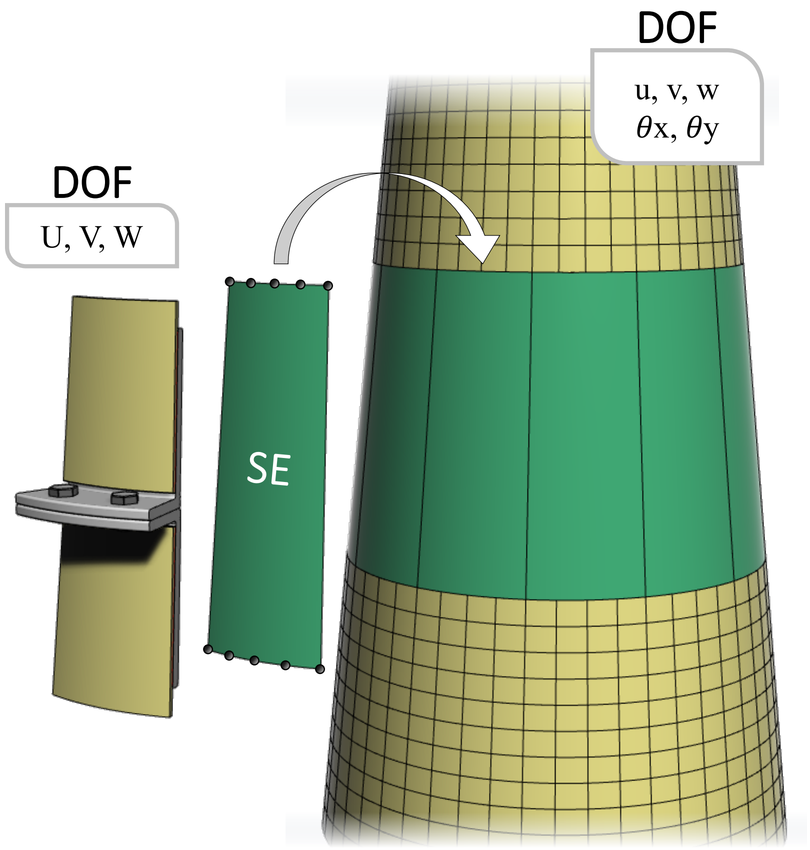
The structure is **divided** into regions of **regular or continuous lamination** and regions of **discontinuity**.

Regular Laminates → Conventional **Shell Model**
Discontinuities → **Condensed** in the form of **ROMs**

The resulting model built with the reduced dimension elements and the **BL** and **SLROM** elements is referred to as Structural Model.



Structural Model
Conventional RM Shell Element
+ SLROM

ROM using Mixing Dimensional Coupling

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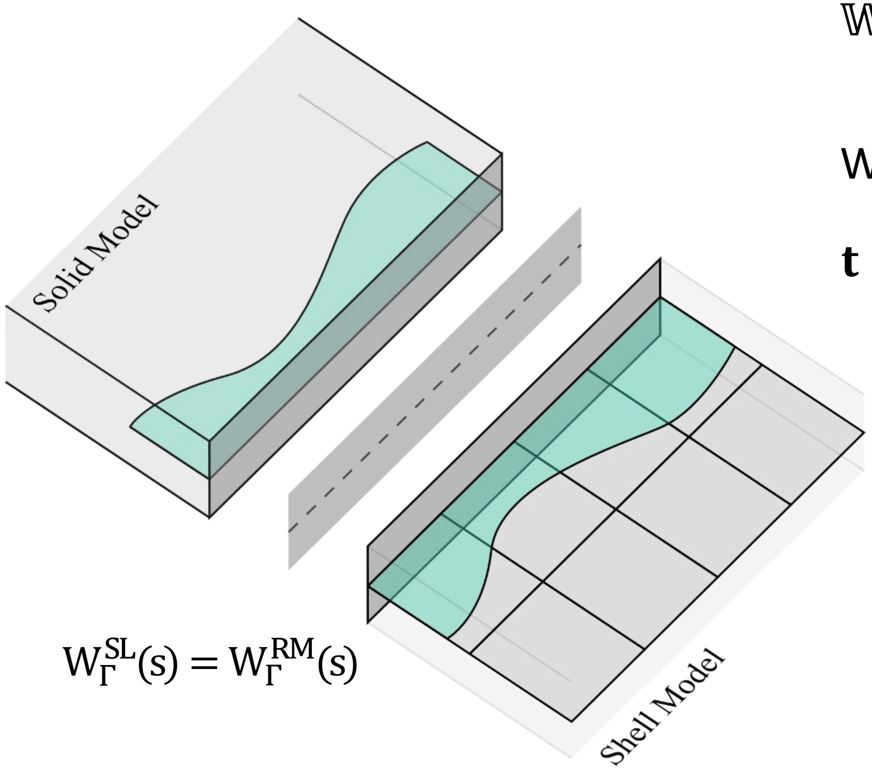
The coupling condition requires the equilibrium of work along the coupling interface between the solid and the shell model. The work is defined as:

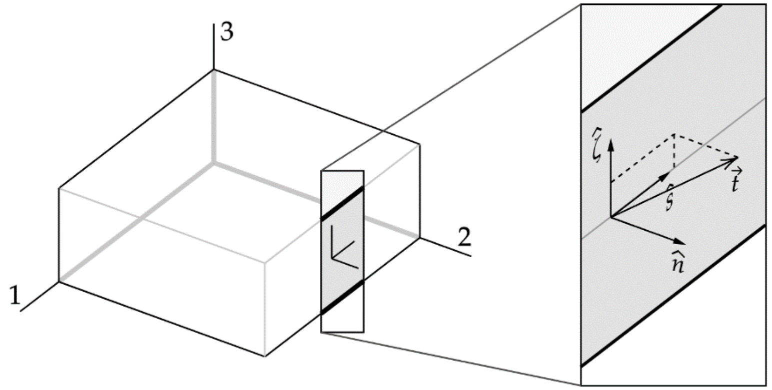


$$W_\Gamma^{SL}(s) = W_\Gamma^{RM}(s)$$

$$\mathbb{W} = \int_A \omega \, dA \qquad \omega = \mathbf{t} \cdot \mathbf{u}$$

When $\hat{n}$ is perpendicular

$$\mathbf{t} = \{\sigma_{nn} \; \sigma_{ns} \; \sigma_{n\zeta}\}$$



And therefore

$$\mathbb{W} = \int_\Gamma \sigma_{nn} u_n \, dA + \int_\Gamma \sigma_{ns} u_s \, dA + \int_\Gamma \sigma_{n\zeta} u_\zeta \, dA$$

$$\mathbb{W}_p + \mathbb{W}_s = \underbrace{\int_\Gamma \{\sigma_{nn} \, \sigma_{ns}\} \cdot \begin{Bmatrix} u_n \\ u_s \end{Bmatrix} dA}_{\textit{in-plane}} + \underbrace{\int_\Gamma \{\sigma_{n\zeta} \, \sigma_{s\zeta}\} \cdot \begin{Bmatrix} u_\zeta \\ 0 \end{Bmatrix} dA}_{\textit{transversal}}$$

ROM using Mixing Dimensional Coupling

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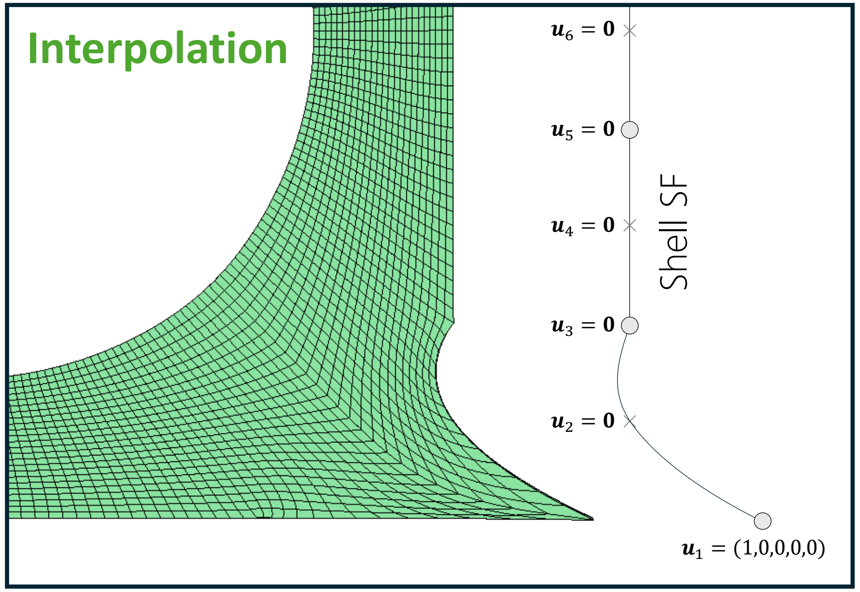
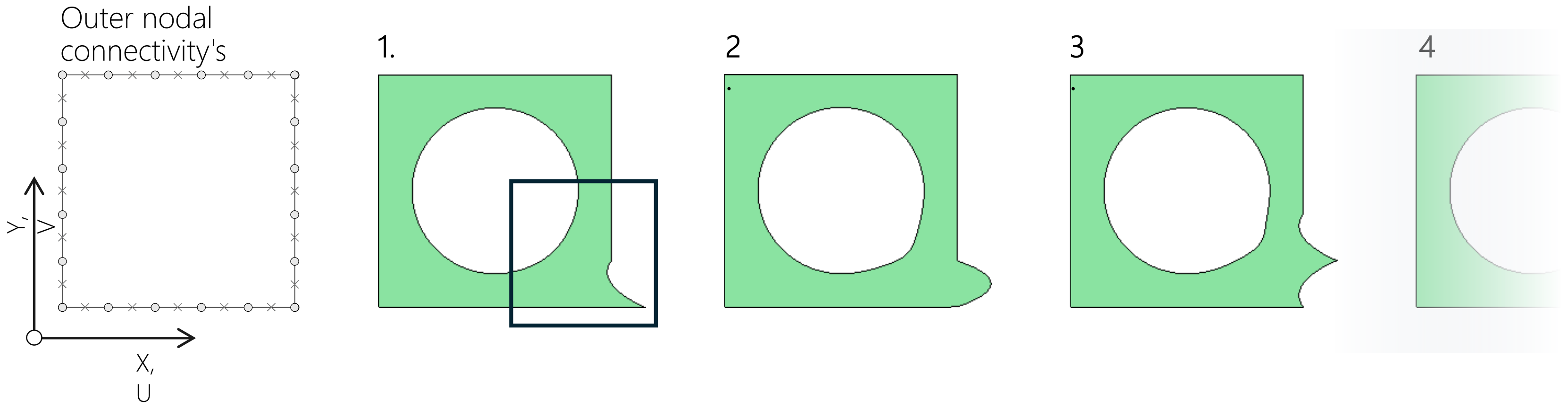
To overcome the **difference** between the **discretization level** on the Solid and Shell models we propose **interpolating** the displacement field of the Shell DOFs along the Solid Interface.

Depending on the continuity a different shape is generated along the interfaces, in the present case a **H1 continuity is used**.





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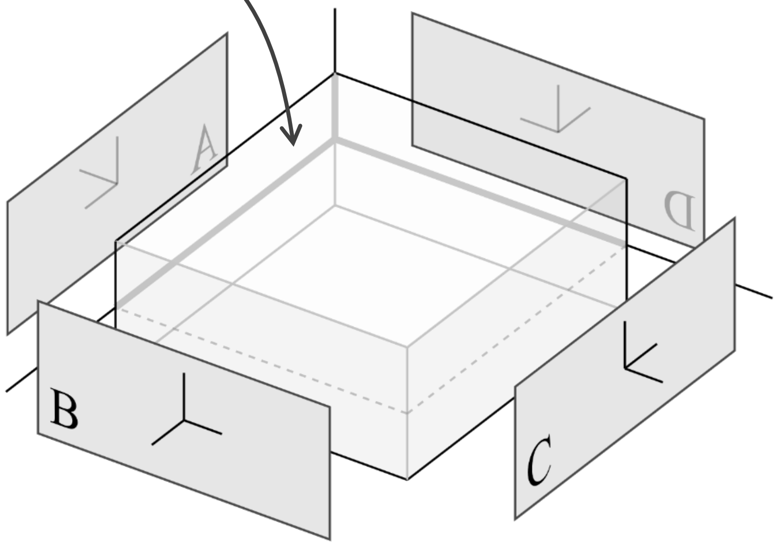
Four **interfaces** are used to **Isolate** a **Solid representation** of the laminate or irregularity.

Solid Model Representation



Number of CC = 7 x # nodes on each interface

$$\mathbf{L}(\mathbf{U},\mathbf{P}_\mathrm{A},\mathbf{P}_\mathrm{B},\mathbf{P}_\mathrm{C},\mathbf{P}_\mathrm{D}) = \Pi_{\Omega_\mathrm{PS}}(\mathbf{U}) + \int \mathbf{P}_\mathbf{A}\cdot\left(\mathbf{u}-\mathbf{c}^\mathbf{T}(\mathbf{U})\right)\mathrm{d}\Gamma + \dots$$

$$\begin{bmatrix} \mathbf{K} & \mathbf{c}_A^\mathrm{T} & \cdots \\ \mathbf{c}_A & \mathbf{0} & \\ \vdots & & \ddots \end{bmatrix} \begin{Bmatrix} \mathbf{U} \\ \mathbf{P}_\mathrm{A} \\ \vdots \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{u}_\mathrm{A} \\ \vdots \end{Bmatrix}$$

$$\mathbf{P} = \begin{Bmatrix} \mathrm{N}_1 \\ \mathrm{N}_{12} \\ \mathrm{M}_1 \\ \mathrm{M}_{12} \\ \mathrm{Q} \\ \varepsilon_{r_s} \\ \kappa_s \end{Bmatrix} \quad \mathbf{u} = \begin{Bmatrix} \mathrm{u_r} \\ \mathrm{v_r} \\ \theta_1 \\ \theta_2 \\ \mathrm{w_r} \\ 0 \\ 0 \end{Bmatrix} \quad \mathbf{c} = \begin{Bmatrix} \mathrm{c}_{\mathrm{N}_1} \\ \mathrm{c}_{\mathrm{N}_{12}} \\ \mathrm{c}_{\mathrm{M}_1} \\ \mathrm{c}_{\mathrm{M}_{12}} \\ \mathrm{c_Q} \\ \mathrm{c}_{\varepsilon_{r_s}} \\ \mathrm{c}_{\kappa_s} \end{Bmatrix} \quad \mathbf{U} = \begin{Bmatrix} \mathrm{U} \\ \mathrm{V} \\ \mathrm{U} \\ \mathrm{V} \\ \mathrm{W} \\ \mathrm{U} \\ \mathrm{V} \end{Bmatrix}$$

ROM using Mixing Dimensional Coupling

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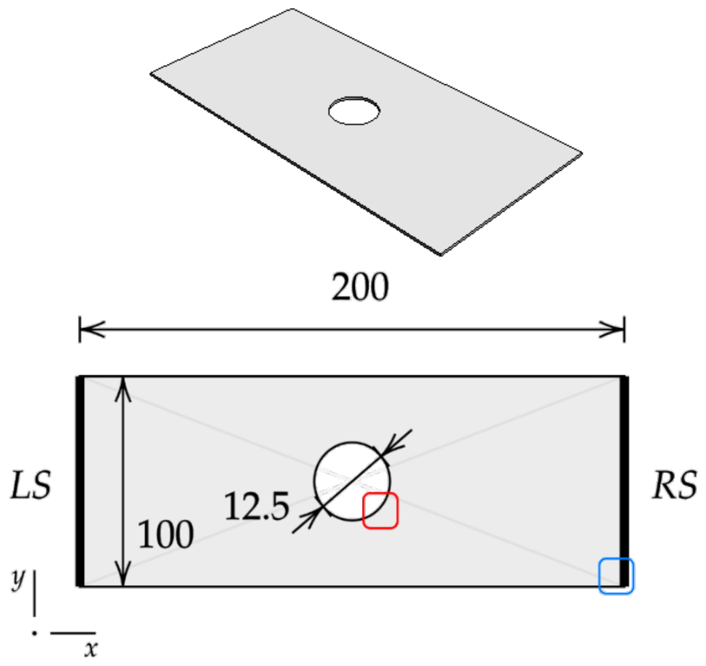
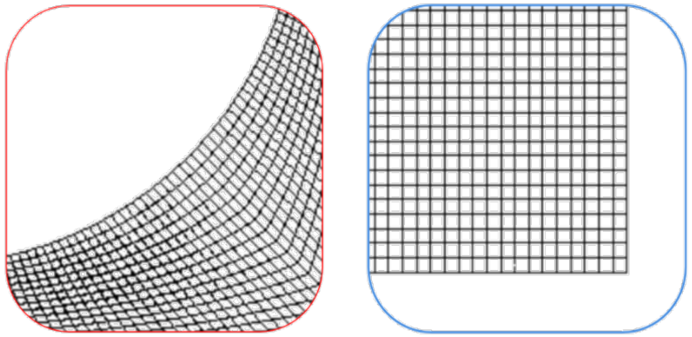
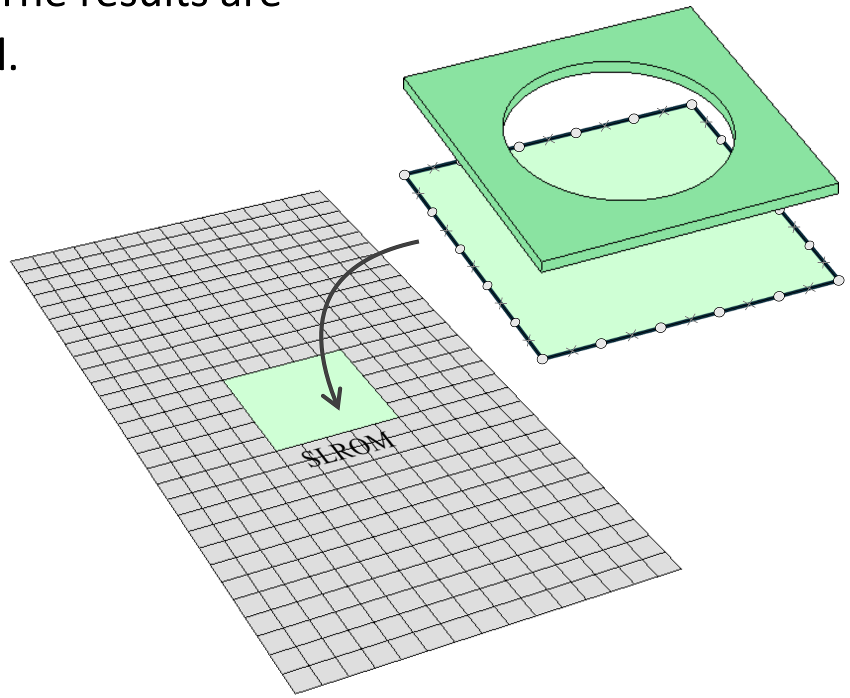
To assess the performance of SLROM defined from **irregular regions** the following Open Hole laminate is studied under traction with a Shell model with a central SLROM. The results are compared with its equivalent **D**irect **N**umerical **S**imulation (**DNS**) Solid model.



Discretization



DNS Solid Model



Structural model assembled with **conventional Shell elements** and a **central SLROM**

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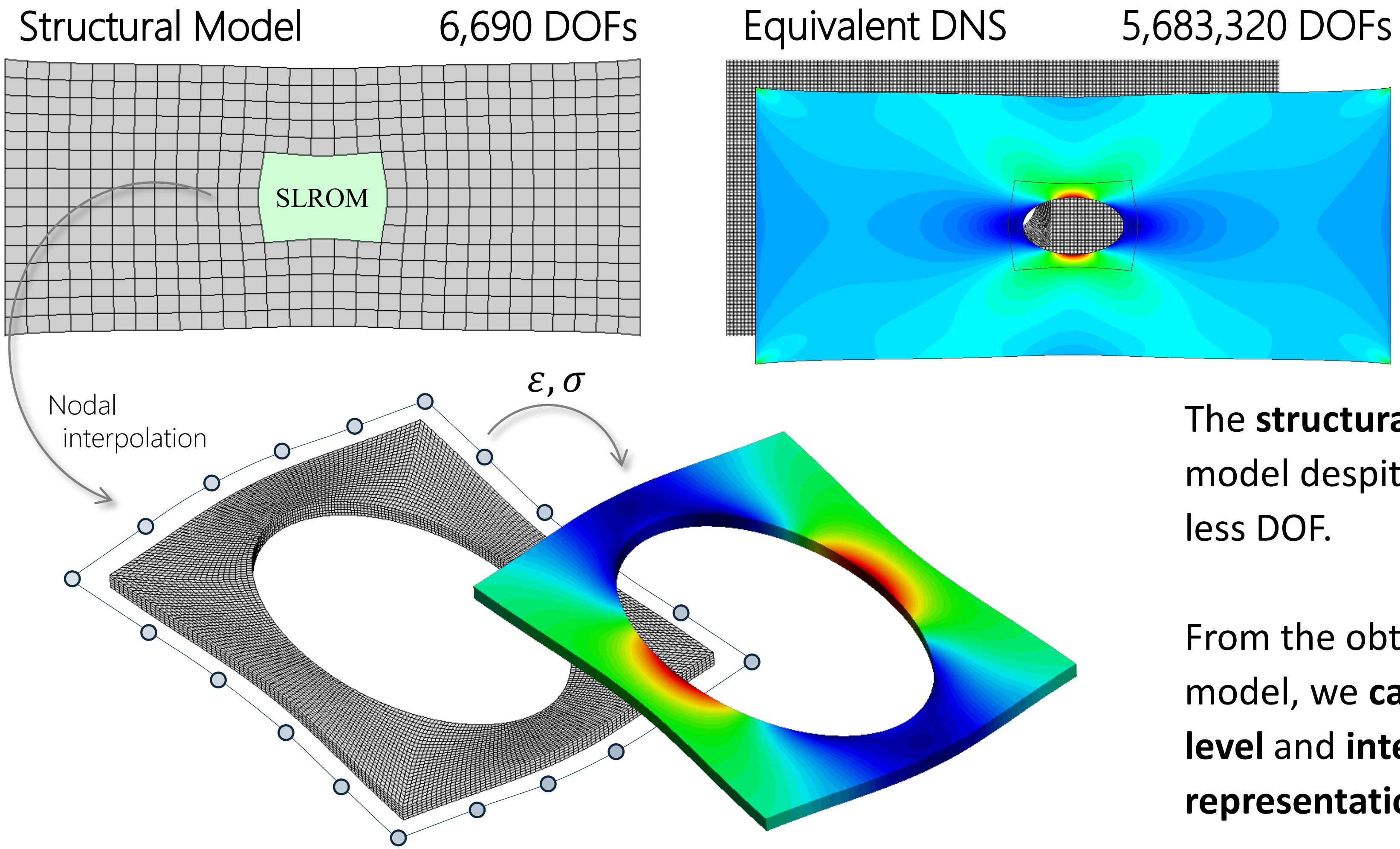

The **structural behavior agrees with the DNS** model despite having 3 orders of magnitude less DOF.

From the obtained deformed structural model, we **can extract the solution at nodal level** and **interpolate it to the solid SLROM representation**.

Conclusions

We have presented two different models for the analysis of large shell structures with irregularities.

- The Multi-scale Approach allows simulating laminates with complex configurations or even small reinforcements

- The ROM using MDC allow replacing a large irregularity in the composite by a super-element that provides the same stiffness to the structure. Afterwards it is possible to analyse the structural element to evaluate its performance.

Acknowledgements

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THANK YOU!

QUESTIONS?

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