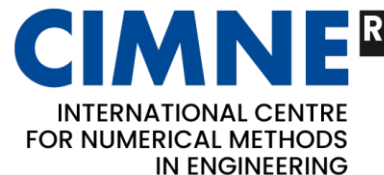


# Topology optimization with additive manufacturing constraints

Authors: **Jose Antonio Torres Lerma**, Dr. Alex Ferrer Ferre and Dr. Fermin Otero Gruer  
**International Centre for Numerical Methods in Engineering**

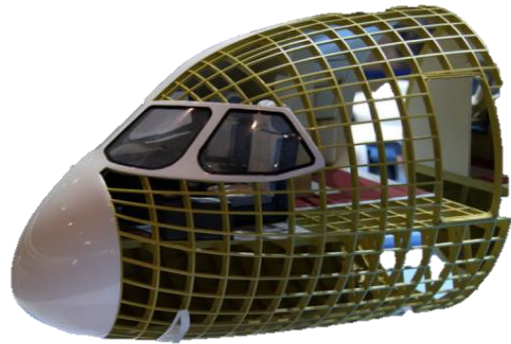


**AMADE Days**  
July 11<sup>th</sup> 2024

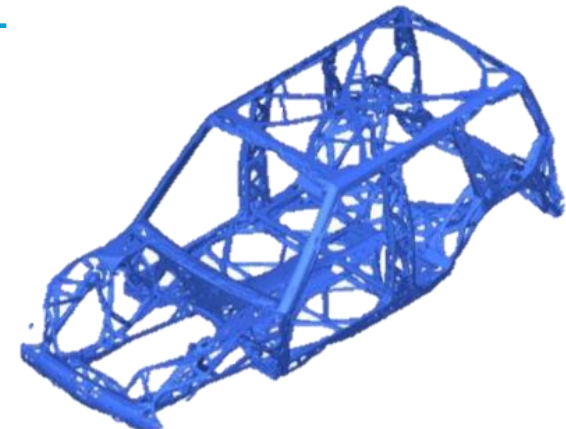
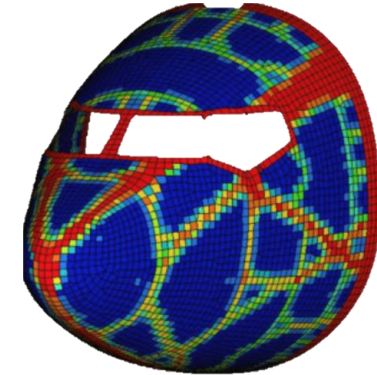
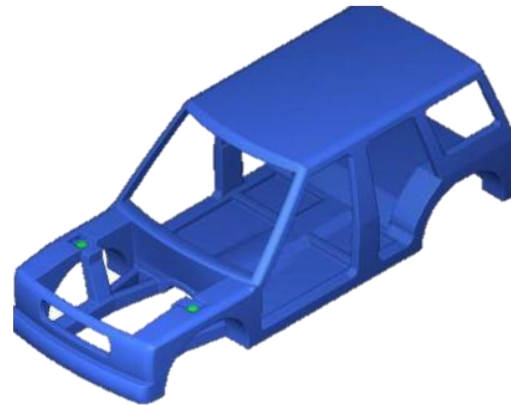
# Background

**Need to reduce fuel consumption and environmental impact in the transport industry**

Aerospace industry

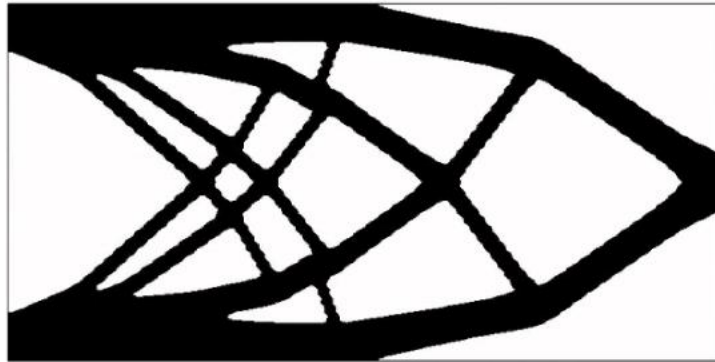


Automotive industry



# Topology Optimization

## Problem formulation



$$\chi = \begin{cases} 1 & \text{if } x \in \Omega \\ 0 & \text{if } x \notin \Omega \end{cases} \quad \begin{cases} \min_{\chi, u} & J(\chi, u) \\ \text{s.t} & a(\chi, u, v) = l(v) \\ & g_i(\chi) \leq 0 \end{cases}$$

Common functionals:

$$J(\chi, u) = \int_D f u(\chi) dV \quad g(\chi) = \int_D \chi dV - V^*$$

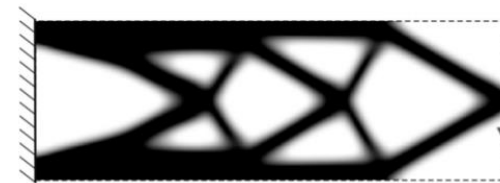
## Design variables



The original design variable is discontinuous



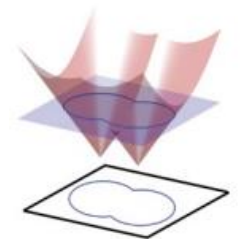
**Density**  $\rho \in [0, 1]$



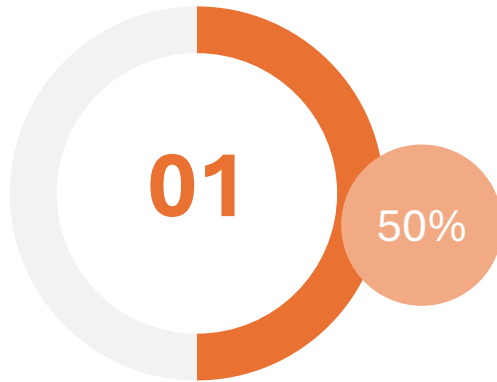
$$C(\rho) = \rho^3 C^{(1)} + (1 - \rho)^3 C^{(0)}$$



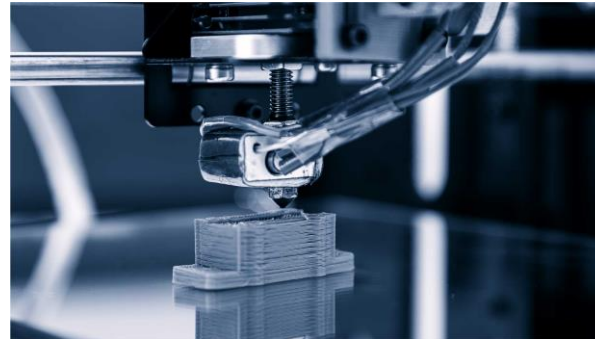
**Level set**  $\chi(\psi) = \begin{cases} 1 & \text{if } \psi \leq 0 \\ 0 & \text{if } \psi > 0 \end{cases}$



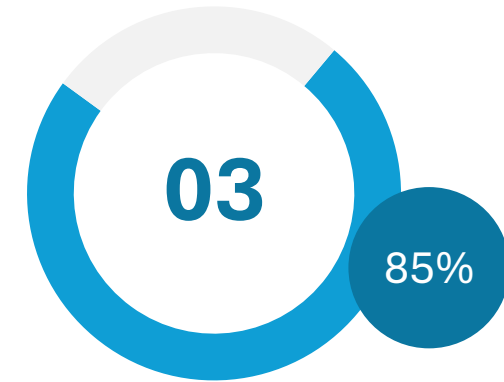
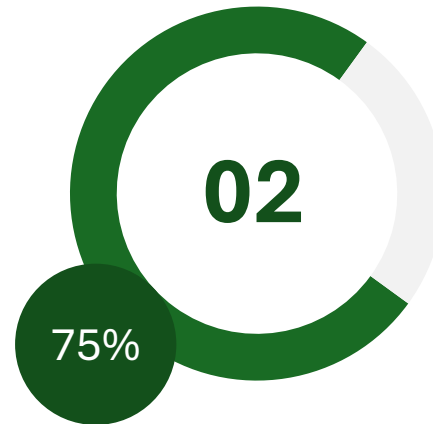
# Additive manufacturing



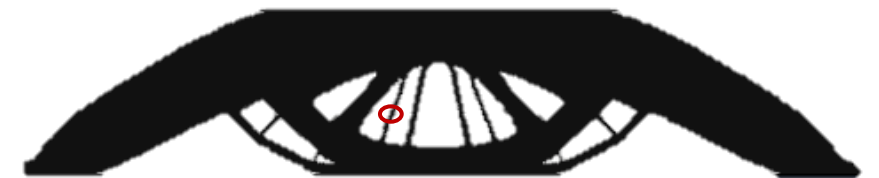
Topology optimized design



Use of additive manufacturing  
(3D CAD data printing)

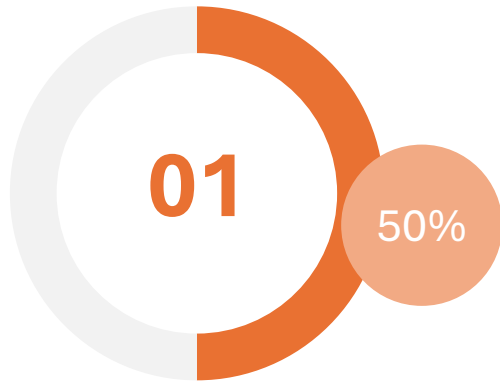


**3D printing process failed!**  
**Not enough sensitivity of the machine**

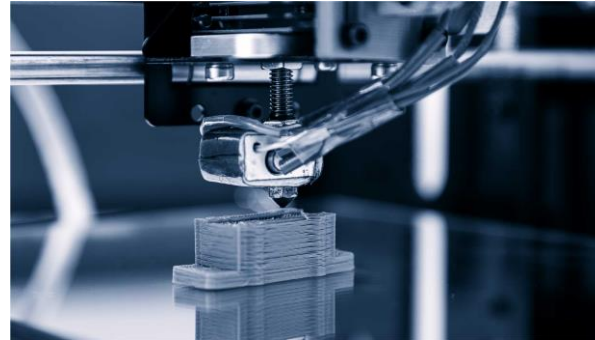


Minimum length scale constraint

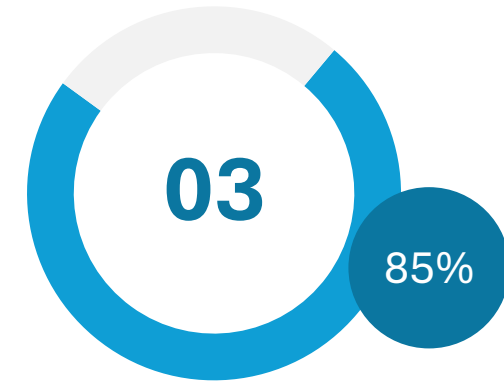
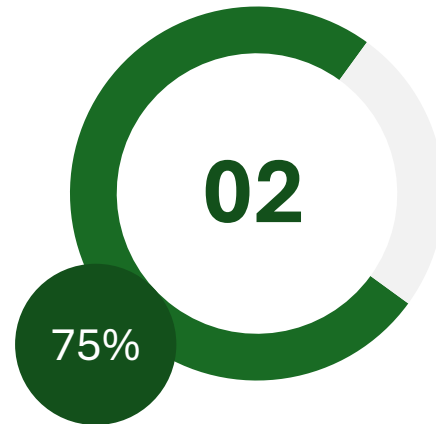
# Additive manufacturing



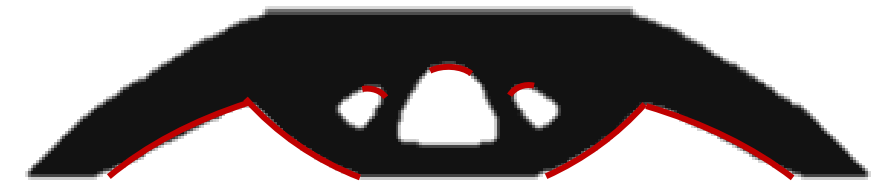
Topology optimized design



Use of additive manufacturing  
(3D CAD data printing)



**3D printing process failed!**  
**Material fell during deposition process** ❌

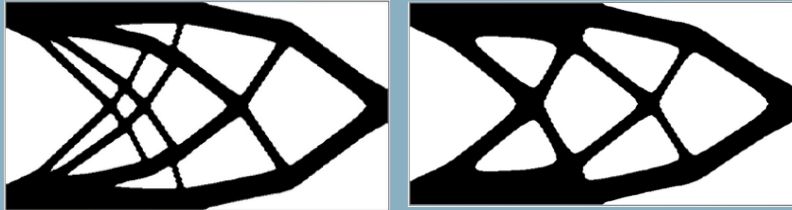


**Overhang constraint**

# Motivation for length scales and overhang

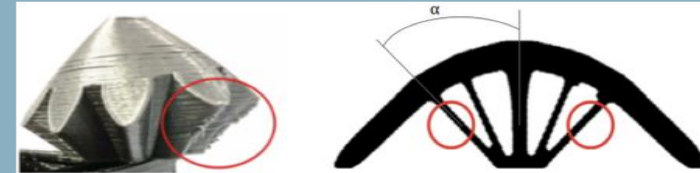
## Length scales constraints

Thickness of bars must be monitored



## Overhang constraints

Angle between the vertical axis and tangent vectors smaller than  $45^\circ$



Bound formulation and three fields representation  
(Lazarov, B.S., F. Wang & O. Sigmund, 2016)

← **Density** →

Overhang filtering through threshold projection  
(Gaynor and Guest, 2016)

Isotropic Perimeter as penalty term  
(S. Amstutz, C. Dapogny & A. Ferrer, 2022)

← **Level set** →

Mechanical constraints (intermediate shapes self weight compliance)  
(G. Allaire et. al., 2017)

# Motivation for length scales and overhang

Current challenges: larger number of inequality & PDE constraints, layer-by-layer computation of the gradient and complex shape derivatives.

## Length scales constraints

Bound formulation and three fields representation

**Isotropic Perimeter as penalty term**

Density



Level set

## Overhang constraints

Overhang filtering through threshold projection

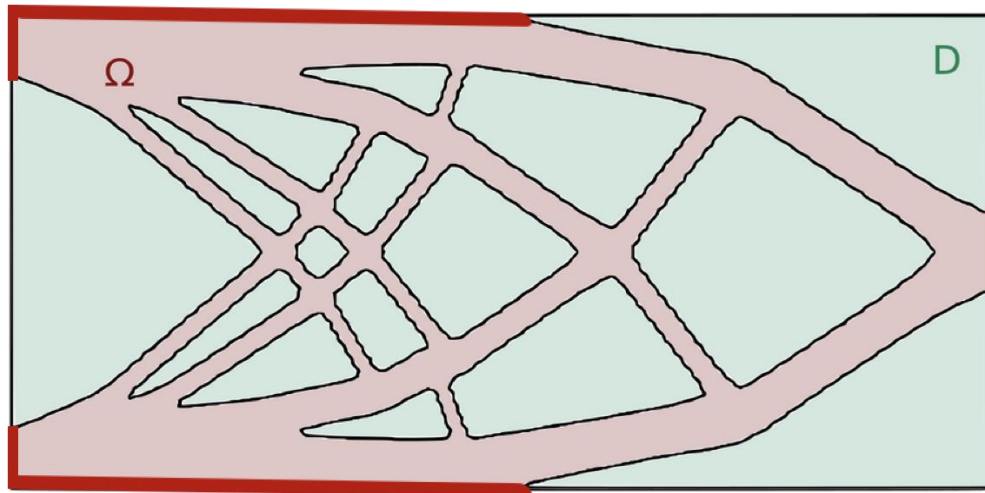
Mechanical constraints (intermediate shapes self weight compliance)

**Aim:** propose a different method to decrease the shape complexity and control the overhang. Advantages: simplicity (perimeter), efficiency (no extra constraints) and useful for density and level set approaches.

# Global isotropic and anisotropic perimeter

## Definition



The **Perimeter** is a functional that computes the length of  $\Omega$  boundaries.



**Relative perimeter**  
INTERNAL BOUNDARIES

**Total perimeter**  
INTERNAL + **EXTERNAL** BOUNDARIES

$$Per(\Omega) = \int_{\partial\Omega} 1d\Gamma$$

}  Shape derivative (level set)  
 Gradient methods (density)

→ **1. Domain filtering**  
**2. Perimeter computation**



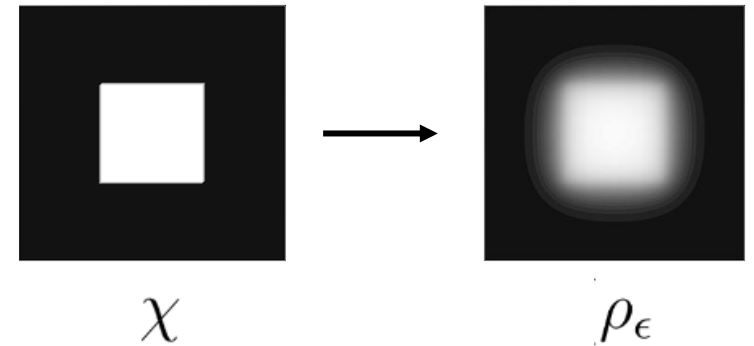
# Global isotropic and anisotropic perimeter

## Definition

### 1. Domain filtering

**Global isotropic relative perimeter:**  $H^1(D)$  projection with Neumann boundary conditions (smoothing).

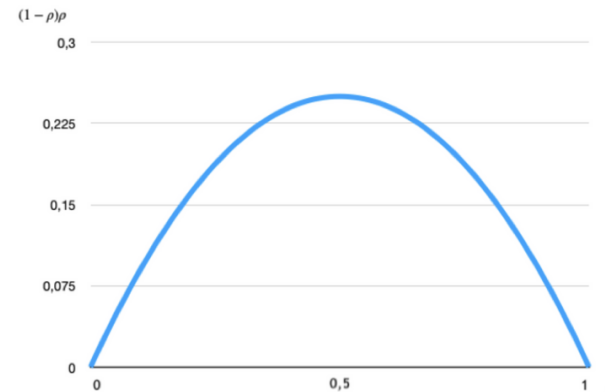
$$\min_{\rho_\epsilon \in H^1} \frac{1}{2} \int_D \underbrace{(\rho_\epsilon - \chi)^2}_{\text{Similar}} dV + \frac{\epsilon^2}{2} \int_D \underbrace{(\nabla \rho_\epsilon)^2}_{\text{Finite gradient}} dV \equiv \min_{\rho_\epsilon \in H^1} J(\rho_\epsilon)$$



### 2. Perimeter computation

$$P = \frac{1}{2\epsilon} \int_D (1 - \rho_\epsilon) \chi \cdot dV$$

(S. Amstutz, C. Dapogny & A. Ferrer, 2022)

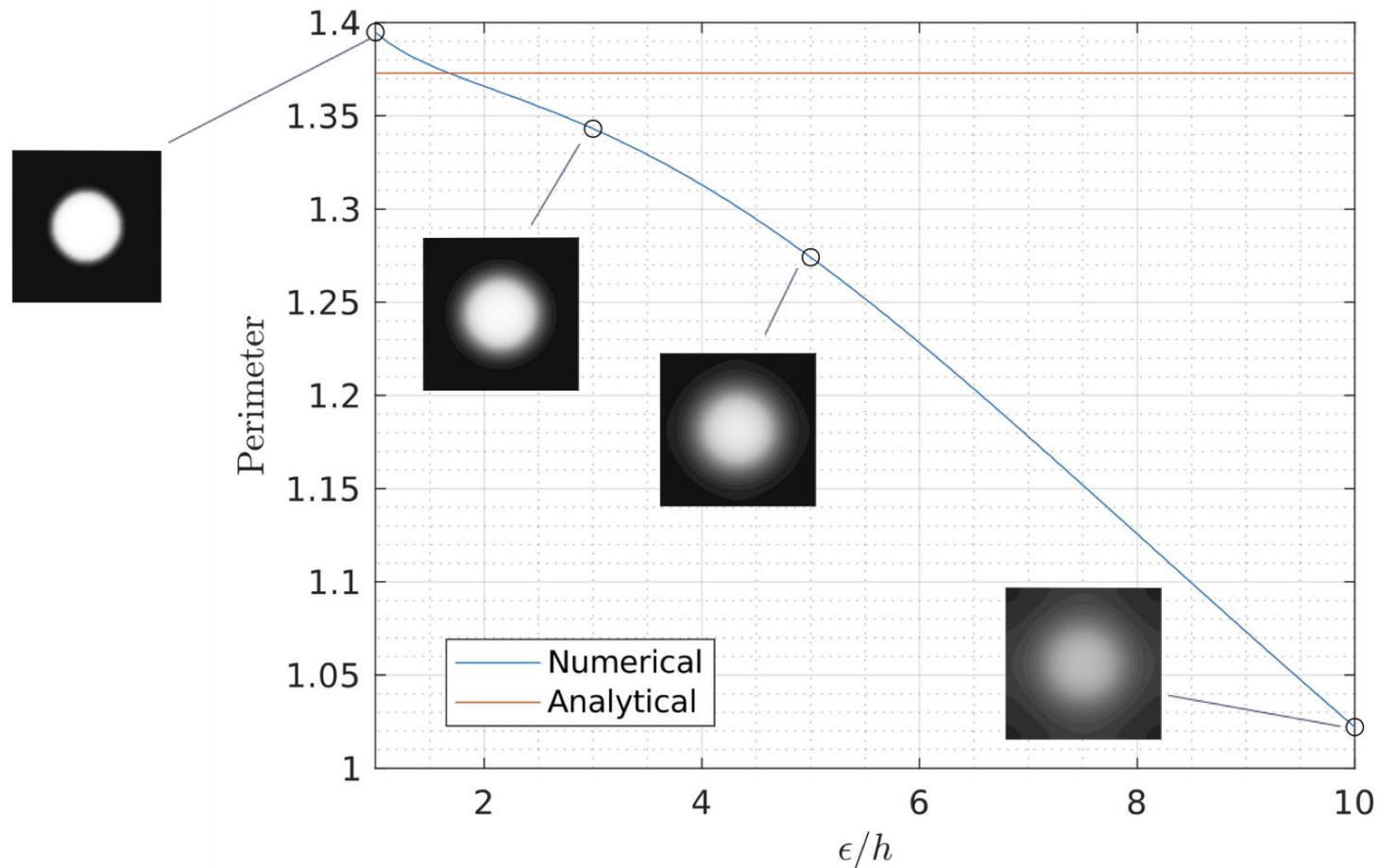


Density example

# Global isotropic and anisotropic perimeter

## Definition

### Relative perimeter convergence



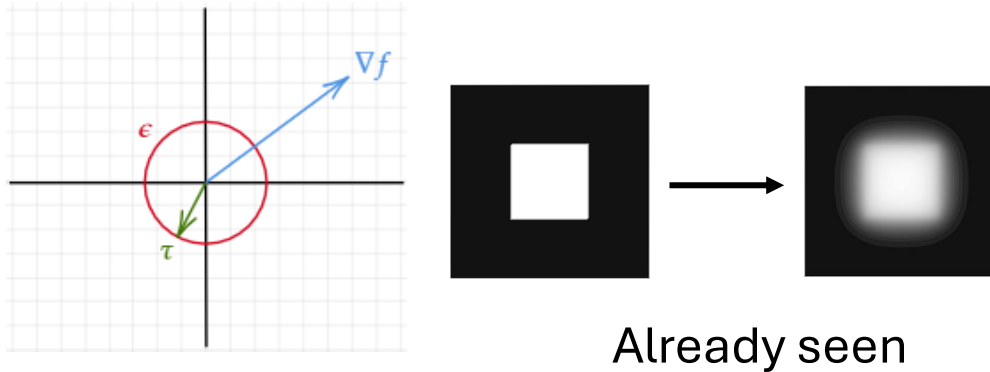
$$P = \frac{1}{2\epsilon} \int_D (1 - \rho_\epsilon) \chi \cdot dV$$

# Global isotropic and anisotropic perimeter

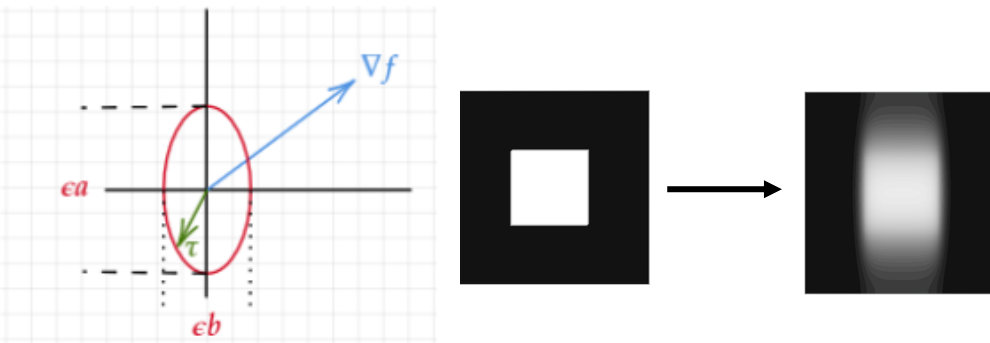
## Definition

### Other domain filterings:

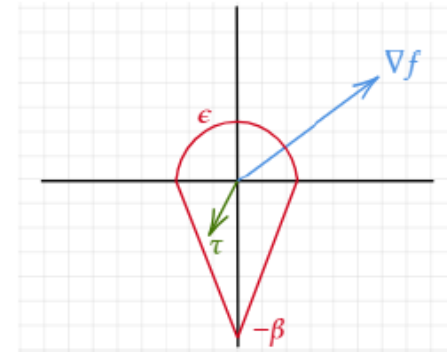
#### 1. Isotropic perimeter.



#### 2. Anisotropic perimeter.



#### 3. Non-linear anisotropic perimeter.



Pending to implement

### Implementation

$$\begin{cases} \min_x & j(u_\chi) + \gamma P(\chi) \\ s.t & \int_D \chi \cdot dV - V^* = 0 \end{cases}$$

# Application to topology optimization of structures and compliant mechanisms

Now, we will solve two optimization problems in the macroscopic scale:

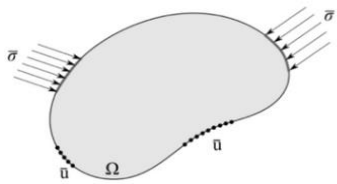
$$\begin{cases} \min_{\chi} & j(u_{\chi}) + \gamma P(\chi) \\ \text{s.t} & \int_D \chi \cdot dV - V^* = 0 \end{cases}$$

Minimum compliance

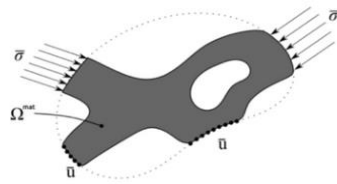
$$j(u_{\chi}) = \int_D f u_{\chi} dV$$

Compliant mechanisms

$$j(u_{\chi}) = \int_{\partial D} k u_{\chi} dV$$



Design domain



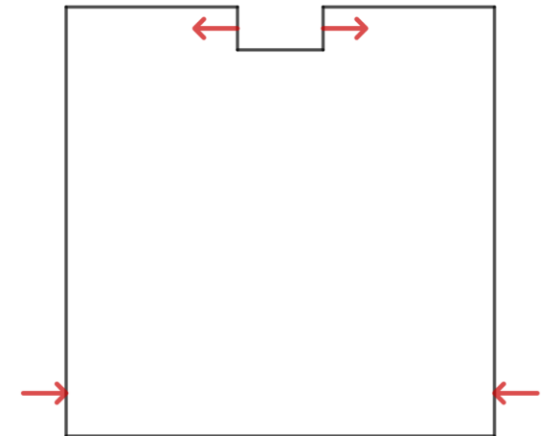
Optimal design

With  $u_{\chi}$  solution of  $a(\chi, u, v) = l(v)$

Several possibilities for the constitutive law

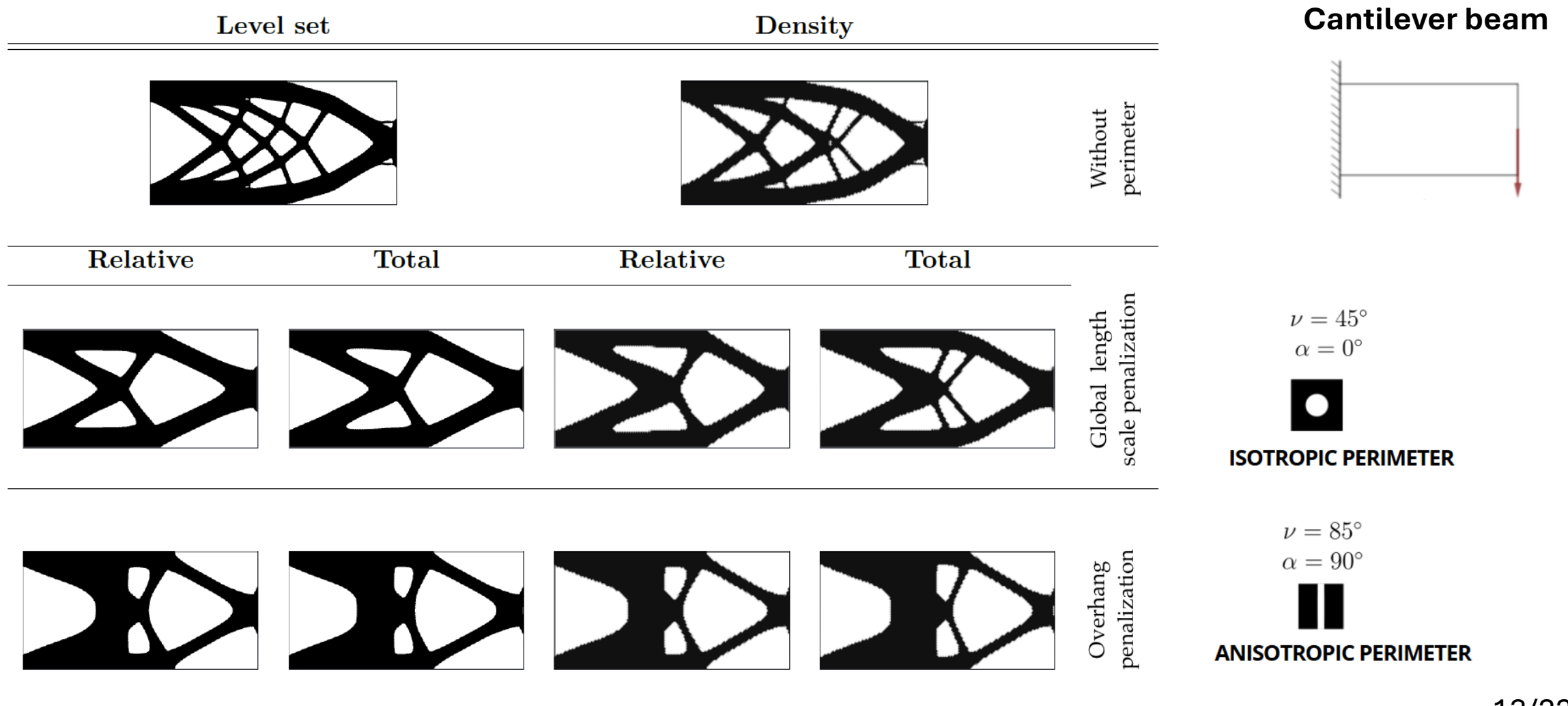


Cantilever beam

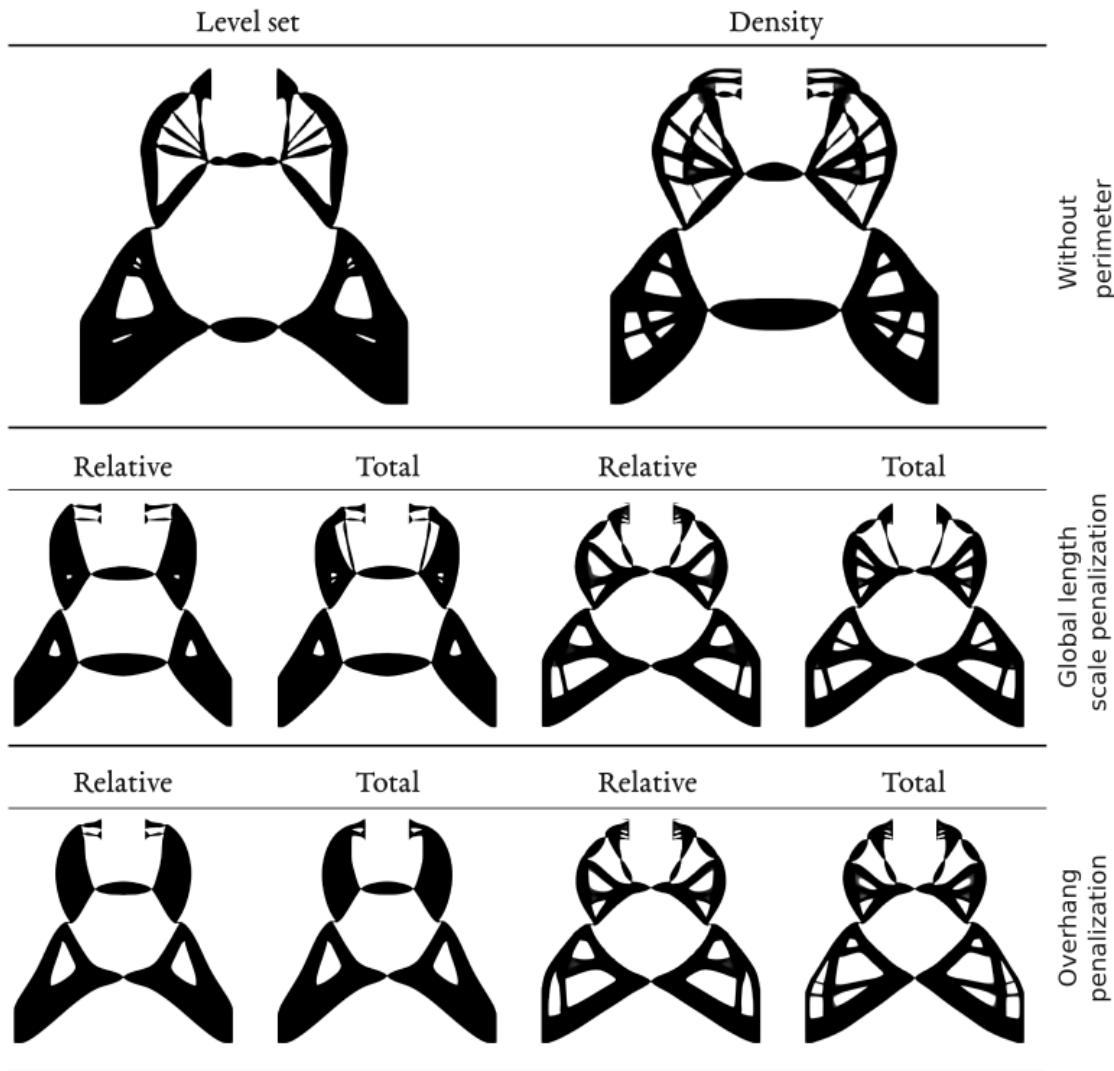


Gripper

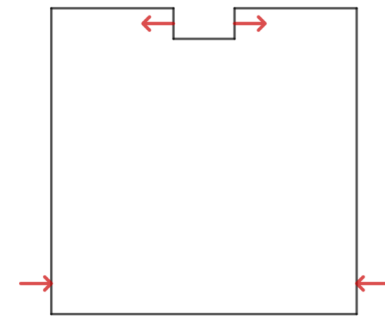
# Application to topology optimization of structures and compliant mechanisms



# Application to topology optimization of structures and compliant mechanisms



Gripper compliant mechanism



$$\nu = 45^\circ$$

$$\alpha = 0^\circ$$



**ISOTROPIC PERIMETER**

$$\nu = 85^\circ$$

$$\alpha = 90^\circ$$



**ANISOTROPIC PERIMETER**



# Extension to material design

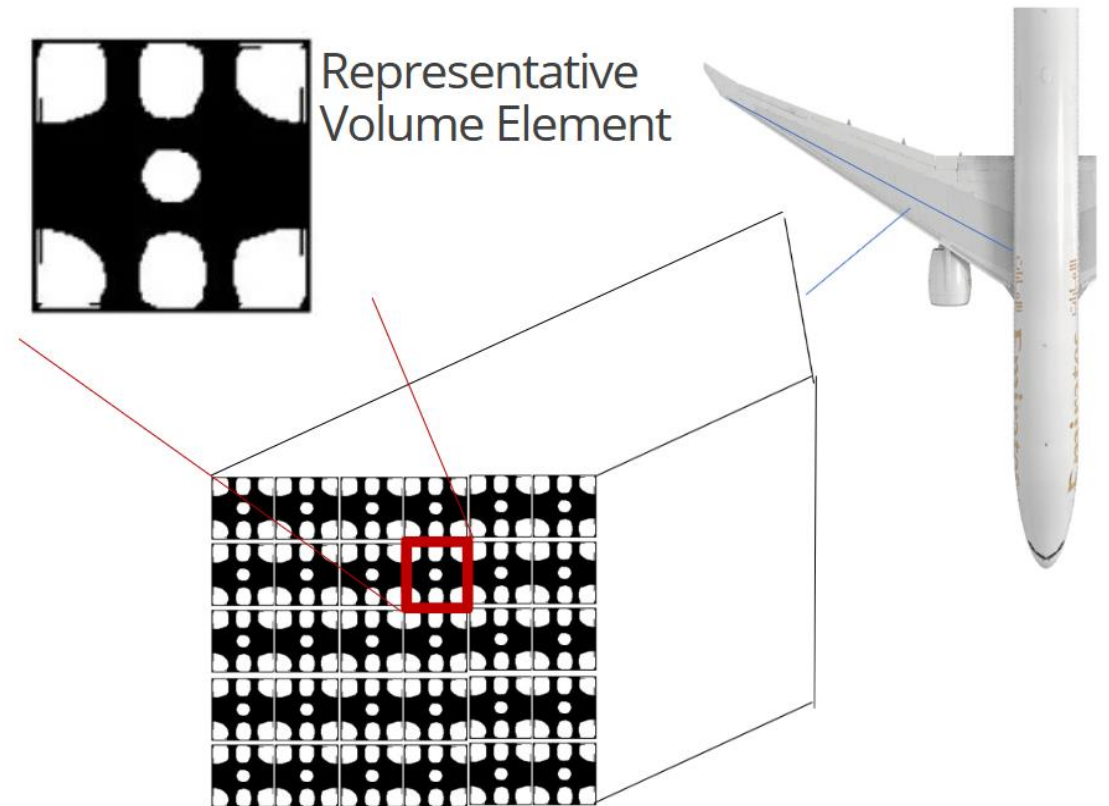
Consider the homogenized elasticity tensor satisfying:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \mathbb{C} \cdot \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix}$$

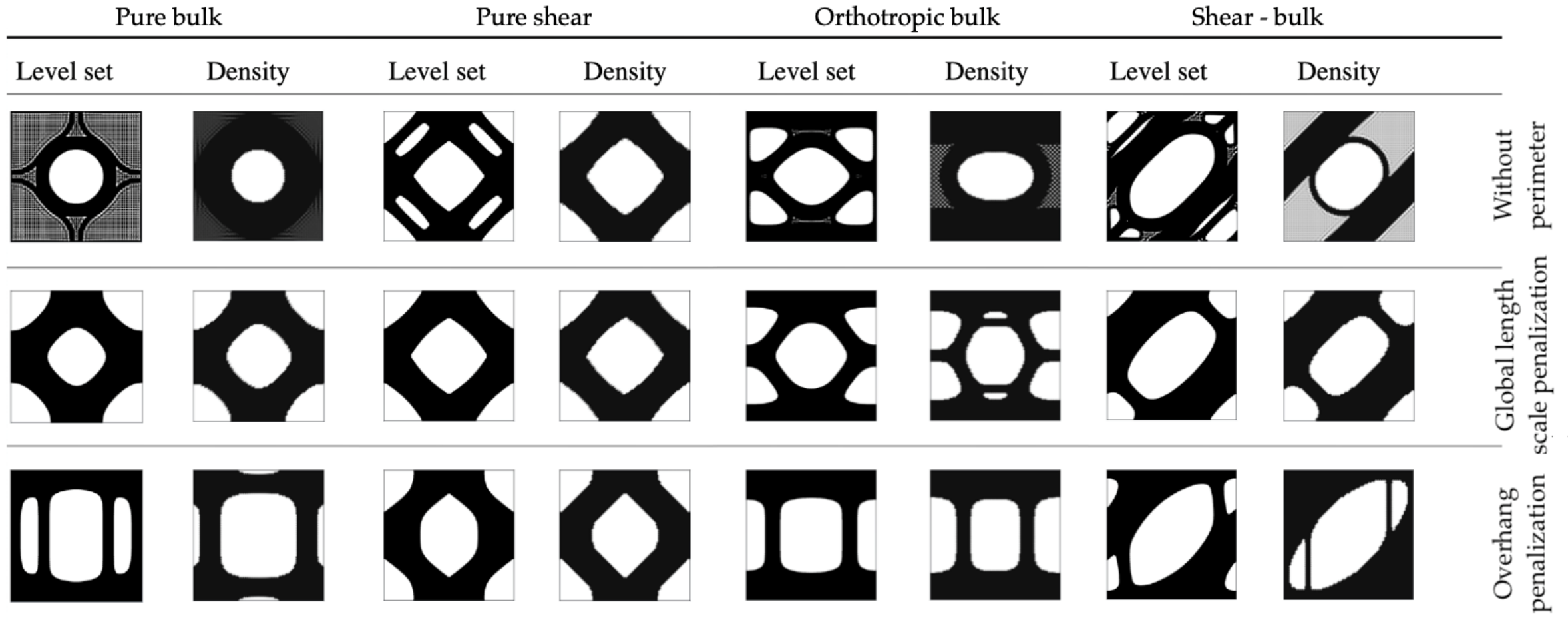
**Optimization problem:**

$$\begin{cases} \min_{\chi} & \alpha_h^T \mathbb{C}^{-1} \beta_h + kP \\ \text{s.t} & \int_D \chi \cdot dV - V^* = 0 \end{cases}$$

(E.A. de Souza Neto, S. Amstutz, S.M. Giusti, A.A. Novotny)



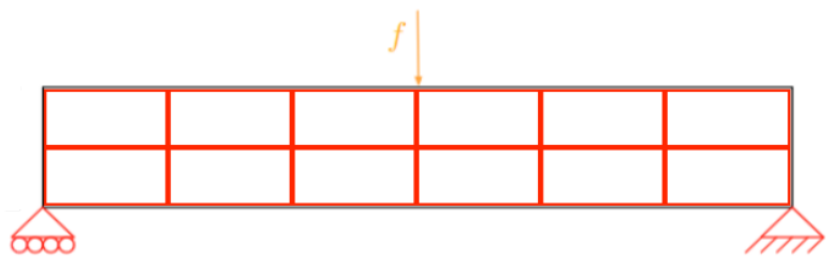
# Extension to material design





# Extension to local perimeter

New motivation: control the length of boundaries and overhang more locally.



One Lagrange multiplier per **subdomain**.

$$\textcircled{1} \begin{cases} \min_{\chi} & \int_D f u_{\chi} dV + \gamma P(\chi) \\ \text{s.t} & \int_D \chi \cdot dV - V^* = 0 \end{cases}$$

$$\textcircled{2} \begin{cases} \min_{\chi} & \int_D f u_{\chi} dV \\ \text{s.t} & \int_D \chi \cdot dV - V^* = 0 \\ & P(\chi) - P^* \leq 0 (\lambda_{Glob}) \end{cases}$$



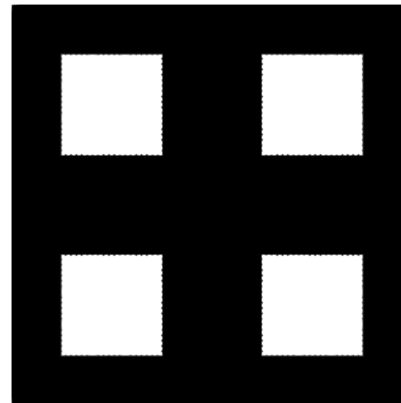
$$\textcircled{4} \begin{cases} \min_{\chi} & \int_D f u_{\chi} dV \\ \text{s.t} & \int_D \chi \cdot dV - V^* = 0 \\ & \frac{1}{2\epsilon} \int_D \delta\lambda(1 - \rho_{\epsilon}(\chi))\chi dV - P^* \leq 0 \end{cases}$$

$$\textcircled{3} \begin{cases} \min_{\chi} & \int_D f u_{\chi} dV \\ \text{s.t} & \int_D \chi \cdot dV - V^* = 0 \\ & P_1(\chi) - P^* \leq 0 (\lambda_1) \\ & P_2(\chi) - P^* \leq 0 (\lambda_2) \\ & \dots \\ & P_{n_{subD}}(\chi) - P^* \leq 0 (\lambda_{n_{subD}}) \end{cases}$$

# Extension to local perimeter

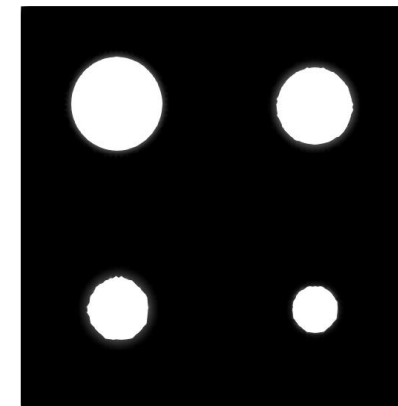
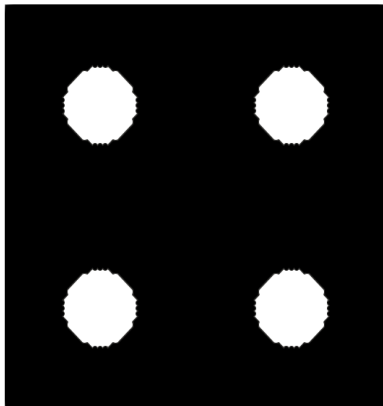
First example: global isotropic relative perimeter versus local isotropic perimeter.

$$\begin{cases} \min_{\chi} & V(\chi) = \int_D \chi \cdot dV \\ \text{s.t} & \frac{1}{2\epsilon} \int_D (1 - \rho_\epsilon(\chi)) \chi dV - P^* \leq 0 \end{cases}$$



Initial guess

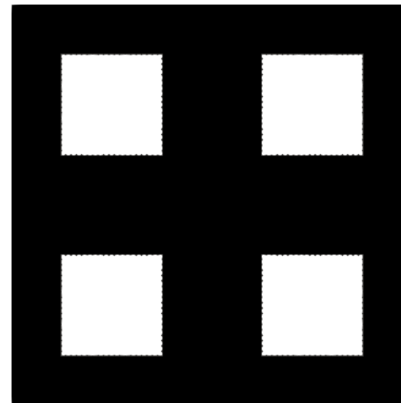
$$\begin{cases} \min_{\chi} & V(\chi) = \int_D \chi \cdot dV \\ \text{s.t} & \frac{1}{2\epsilon} \int_D \delta\lambda (1 - \rho_\epsilon(\chi)) \chi dV - P^* \leq 0 \end{cases}$$



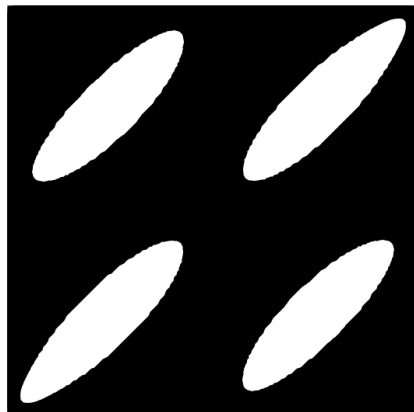
# Extension to local perimeter

Second example: global anisotropic relative perimeter versus local anisotropic perimeter.

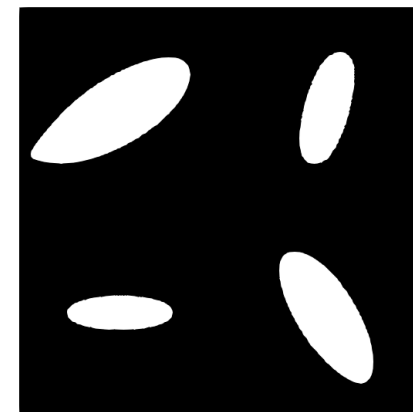
$$\begin{cases} \min_{\chi} & V(\chi) = \int_D \chi \cdot dV \\ \text{s.t} & \frac{1}{2\epsilon} \int_D (1 - \rho_{\epsilon}(\chi)) \chi dV - P^* \leq 0 \end{cases}$$



Initial guess



$$\begin{cases} \min_{\chi} & V(\chi) = \int_D \chi \cdot dV \\ \text{s.t} & \frac{1}{2\epsilon} \int_D \delta\lambda (1 - \rho_{\epsilon}(\chi)) \chi dV - P^* \leq 0 \end{cases}$$



# Conclusions

Main idea: smooth the boundary & penalize gray areas

- Isotropic smoothing controls the global length
- Anisotropic smoothing may generally penalize also overhang regions

Avoiding bound formulation or extra mechanical constraints

Method useful for density and level set

Ongoing:

- Local perimeter as constraints
- Implementing the sense of the 3D printing
- **Topology optimization with composites**



SwanLab/Swan

<https://github.com/SwanLab/Swan/>

WHAT

## Features

### Material design

Swan can perform analyses of microstructures: when combining it with its topology optimization capabilities, **novel metamaterials** can be designed in order to tackle complex challenges and further push the boundaries of engineering.

### Structural design

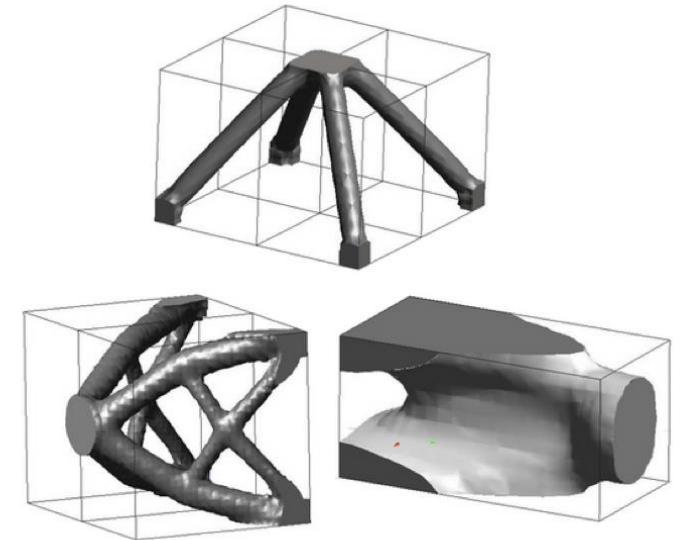
The modular design of Swan allows the combination of several functionals in order to **define complex optimization problems**. Among the functionals that can be used as constraints are compliance, volume, and perimeter. Swan also features **density-based optimizers** like Projected Gradient, MMA and IPOPT, as well as **level-set methods** such as SLERP, Projected SLERP and Hamilton-Jacobi.

### Multi-scale

One of the key features that sets Swan apart from other topology optimization toolboxes is the ability to design **optimal materials at the micro scale**, and reuse the obtained results to perform analyses at the **macro level**.

### Multiphysics, and much more

We are constantly looking ahead and recruiting new contributors in order to keep expanding Swan's capabilities. Among the planned upcoming features are **multiphysics**, 3D microstructural optimization, and many more.



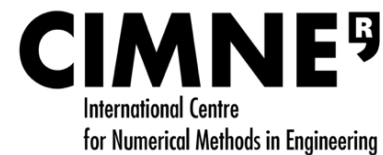
Optimization of the 3D cantilever benchmark

## Global length and overhang control for level set and density approaches via perimeter minimization

Jose Torres

Fermin Otero

Alex Ferrer



Article already submitted  
to a journal!

[jatorres@cimne.upc.edu](mailto:jatorres@cimne.upc.edu)



<https://github.com/SwanLab/Swan/>