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Toward the prediction of the fatigue lifetime of laminated composites, using an incremental damage model with observable variables

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❖ **A main industrial challenge:**

- Efficiently designing lighter structures,
- Ensuring optimum performances and safety,
- Being competitive

➢ An alternative to metals: composite materials

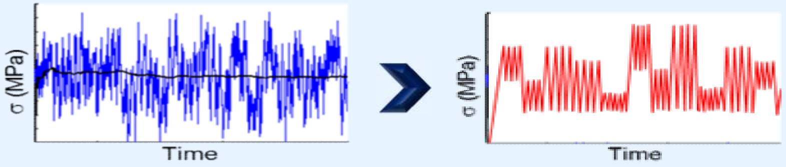
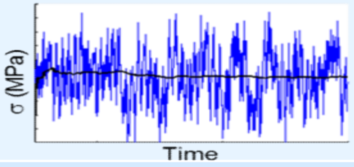
Necessity of **dimensioning in fatigue** and developing **predictive models**

❖ **Advantages of such a model**

- Cut experimental costs,
- Reduce design delays,
- Enable the use of appropriate safety coefficients

❖ **Objectives:**

- ➢ Propose a simple damage model able to predict the crack density both in static and in fatigue,
- ➢ Develop a model based on existent data (T700GC/M21),
- ➢ Validate the approach with experimental tests on a currently in-use generation material (IMA/M21ev)

	❖ Cyclic fatigue damage models	❖ Time based damage models
	➢ Real loading is approximated by a cyclic loading  	➢ Real loading is applied, ➢ **All the cycles** are simulated 
Internal variables	❖ **Modifying damage parameters law $\boldsymbol{d = f\left(\sigma, \alpha_j(N)\right)}$** *[Talreja 1992], [Thionnet 2002], [Revest 2011], [Carraro 2017]* ❖ **Considering two different evolution laws** *[Payan 2002], [Hochard 2006], [Rakatoarisoa 2013]* ❖ **Constitutive Damage Model** *[Maimí 2007], [Llobet 2020], [Carraro 2021]*	❖ **Metallic materials** *[Lemaitre 1992], [Cantournet 2002]* ❖ **Composite Materials** *[Talreja 1999], [Angrand 2016], [Sally 2020]*
Observable variables	❖ **Matrix cracking (static and fatigue)** *[Llobet 2018]*	❖ **Static** *[Germain 2020]* ❖ **Fatigue** *Nothing yet to our knowledge*

❖ EXPERIMENTAL CAMPAIGN

Tensile test experimental set up on the IMA/M21ev

❖ Materials

- **T700GC/M21** : thermoset, epoxy and continuous carbon fibers
- **IMA/M21ev** : thermoset, epoxy and continuous carbon fibers, reinforced at the interface

❖ Tests performed

- Quasi-static tensile tests with monitoring levels,
- Fatigue, 100 000 cycles at $f = 5\ Hz$ and $R = 0{,}05$ (IMA/M21ev)
- Fatigue, 100 000 cycles at $f = 5\ Hz$ and $R = 0{,}1$ (T700GC/M21)

❖ Instrumentation

- Acoustic emission,
- Digital Images Correlation,
- Optic microscopy

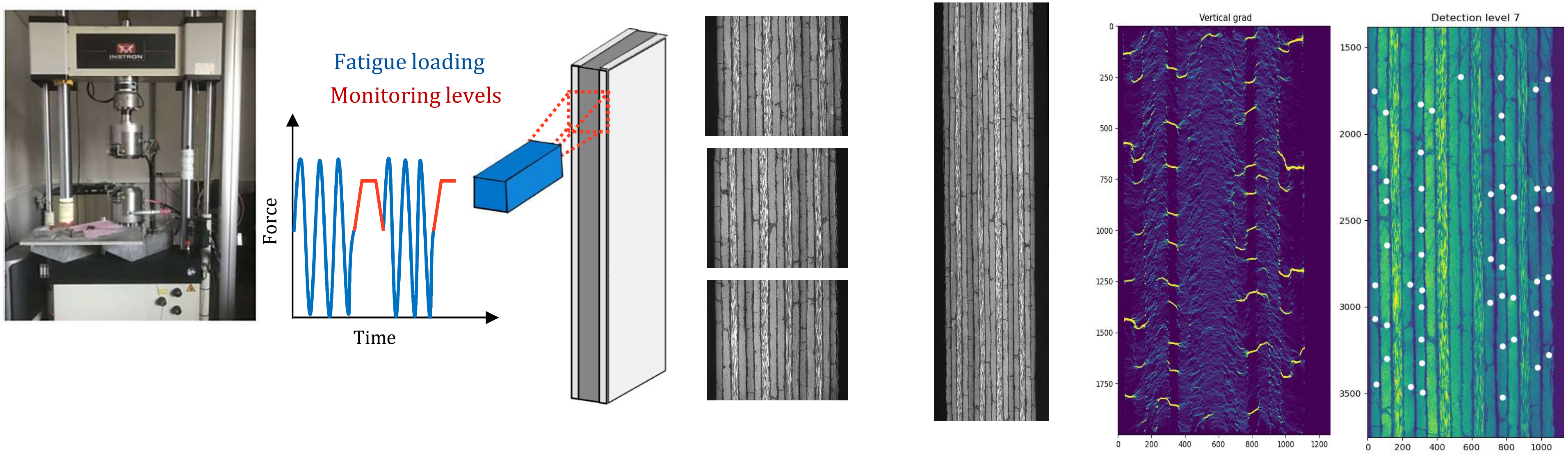
❖ AUTOMATED CRACK DETECTION TOOL

Microscopic takes on the specimen's edge

Reconstitution

Displacement gradients

Crack detection



Working principle of the microscopic takes for the automatic crack count on the IMA/M21ev

❖ **Normalized crack density**: $\bar{\rho} = \frac{N_{cracks}}{L_{obs}} h$

❖ OPERATION PRINCIPLE

❖ Laminate
- Macroscopic scale,
- Σ load

❖ Unidirectional Ply (UD)
- Mesoscopic scale,
- Calculation of σ_k and ε_k,
- Behavior law

❖ **Residual thermal stress:** refined damage onset

$$\varepsilon_{th}^{mat} = \begin{bmatrix} \alpha_L \\ \alpha_T \\ \alpha_{LT} \end{bmatrix} (T - T_0) \longrightarrow$$

❖ Rotation matrix
❖ Behavior law
❖ $N_{th,k}^{init} = (h_{k+1} - h_k)\sigma_{th,k}^{specimen}$
$M_{th,k}^{init} = (h_{k+1}^2 - h_k^2)\sigma_{th,k}^{specimen}$

$$\longrightarrow \begin{bmatrix} N_{th}^{final} \\ M_{th}^{final} \end{bmatrix}_k = \begin{bmatrix} N_{th\,I}^{init} + \Sigma_I e \\ N_{th\,II}^{init} + \Sigma_{II} e \\ N_{th\,III}^{init} + \Sigma_{III} e \\ M_{th\,I} + 0 \\ M_{th\,II} + 0 \\ M_{th\,III} + 0 \end{bmatrix}_k \xrightarrow{\text{Constitutive equations}} \sigma_{th} = Q : \underbrace{(\varepsilon - \varepsilon_{th})}_{\varepsilon^*}$$

Thermal part
Mechanical part

α : Thermal dilatation coefficient in longitudinal and transverse direction
T : Test temperature
T_0 : Identified stress-free temperature
N_{th}^{init} and M_{th}^{init} : Calculated with ε_{th}
Σ : Macroscopic stress applied to the laminate
e: Total laminate thickness

❖ **Damage driving force:** $y_k = \frac{1}{2}\left[\varepsilon_{2,k}^{*+2} Q_{22,k}^{mat} + a_{66} Q_{66,k}^{mat} \varepsilon_{6,k}^{*+2}\right]$

Q^{mat} : Elastic stiffness tensor,
a_{66} : Parameter to identify,
$^{+}$: Positive part of the strain tensor [Ju 1989]

❖ OPERATION PRINCIPLE

❖ Laminate
- Macroscopic scale,
- Σ load

❖ Unidirectional Ply (UD)
- Mesoscopic scale,
- Calculation of σ_k and ε_k,
- Behavior law

❖ **Damage driving force**: $y_k = \frac{1}{2}\left[\varepsilon^{*+2}_{2,k} Q^{mat}_{22,k} + a_{66} Q^{mat}_{66,k} {\varepsilon^{*+}}^2_{6,k}\right]$

❖ **Ply thickness effect**
- ➢ Saturation value $\bar{\rho}_c$,
- ➢ Kinetic y^c_{stat} and p_{stat},
- ➢ Threshold $y^0_{stat} = \frac{1}{2}\left(Y^{is+2}_{\varepsilon\,22} Q^{mat}_{22}\right)$

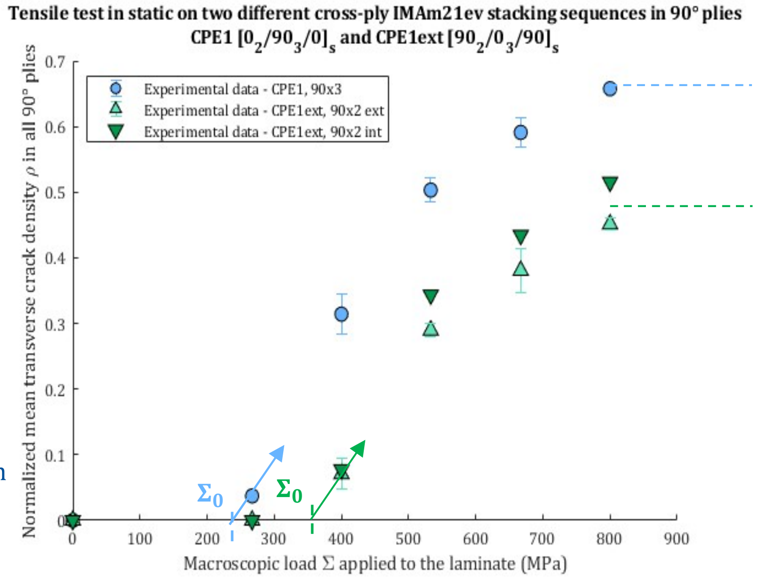
with $Y^{is}_{\varepsilon\,22} = \max\left(\boldsymbol{\varepsilon}^{Re}_{22}, \boldsymbol{\varepsilon}^{R}_{22}\right)$

$$= \max\left(\frac{1}{Q^{mat}_{22}}\sqrt{\frac{8G_{Ic}}{\pi h \Lambda^0_{22}}} - \frac{Q^{mat}_{12}}{Q^{mat}_{22}}\boldsymbol{\varepsilon}^{*}_{12}, \boldsymbol{\varepsilon}^{R}_{22}\right) \text{ Inner ply}$$

[*Camanho 2005*]
[*Dvorak 1987*]

$$\text{or} = \max\left(\frac{1}{Q^{mat}_{22}}\sqrt{\frac{4G_{Ic}}{\pi h \Lambda^0_{22}}} - \frac{Q^{mat}_{12}}{Q^{mat}_{22}}\boldsymbol{\varepsilon}^{*}_{12}, \boldsymbol{\varepsilon}^{R}_{22}\right) \text{ Outer ply}$$

$Y^{is}_{\varepsilon\,22}$: In situ strength written in strain,
Q^{mat} : Rigidity matrix,
ε^{Re}_{22} : Energy criterion,
ε^{R}_{22} : Strain criterion,
G_{Ic} : Critical energy restitution rate in mode I
h : Ply thickness,
Λ^0_{22} : Inverse of Q_{22}



❖ OPERATION PRINCIPLE

❖ Laminate
- Macroscopic scale,
- Σ load


Scale change

❖ Unidirectional Ply (UD)
- Mesoscopic scale,
- Calculation of σ_k and ε_k,
- Behavior law

❖ **Damage driving force**: $y_k = \frac{1}{2}\left[\varepsilon_{2,k}^{*+2} Q_{22,k}^{mat} + a_{66} Q_{66,k}^{mat} \varepsilon^{*+}{}_{6,k}^{2}\right]$

Q^{mat} : Elastic stiffness tensor,
a_{66} : Parameter to identify,
$^{+}$: Positive part of the strain tensor [Ju 1989]

❖ **Normalized crack density**: $\bar{\rho} = \frac{N_{cracks}}{L_{obs}} h$

❖ Generalized **cracking kinetics**: $\dot{\bar{\rho}} = \underbrace{f_{stat}(\bar{\rho}) g_{stat}(y) \dot{y}_{max}}_{\text{Static}} + \underbrace{f_{cycl}(\bar{\rho}) g_{cycl}(y) [\langle \dot{y} \rangle_{+} - \dot{y}_{max}]}_{\text{Fatigue}}$

$$f_{stat}(\bar{\rho}) = (\bar{\rho}_c - \bar{\rho})$$

$$g_{stat}(y) = \frac{p_{stat}}{2\sqrt{y_{stat}^{c}}\sqrt{y_{max}}} \left\langle \frac{\sqrt{y_{max}} - \sqrt{y_{stat}^{0}}}{\sqrt{y_{stat}^{c}}} \right\rangle_{+}^{p_{stat}-1}$$

4 Static parameters

$$f_{cycl}(\bar{\rho}) = \bar{\rho}_c \left(1 - \frac{\bar{\rho}}{\bar{\rho}_c}\right)^{n_{cycl}}$$

$$g_{cycl}(y) = \frac{p_{cycl}}{2\sqrt{y_{cycl}^{c}\sqrt{y}}} \left\langle \frac{\sqrt{y} - \sqrt{y_{cycl}^{0}}}{\sqrt{y_{cycl}^{c}}} \right\rangle_{+}^{p_{cycl}-1}$$

4 Fatigue parameters

❖ OPERATION PRINCIPLE

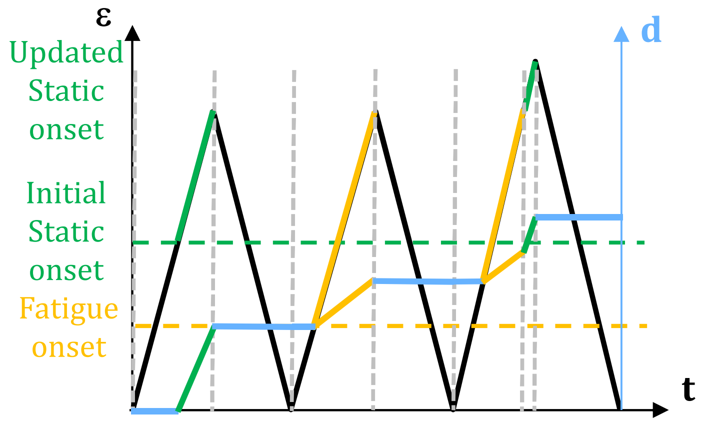
❖ Generalized **cracking kinetics**: $\dot{\bar{\rho}} = \underbrace{f_{stat}(\bar{\rho})g_{stat}(y)\dot{y}_{max}}_{\textbf{Static}} + \underbrace{f_{cycl}(\bar{\rho})g_{cycl}(y)[\langle\dot{y}\rangle_+ - \dot{y}_{max}]}_{\textbf{Fatigue}}$

❖ Presentation of the material approach

- ➢ One unique damage variable for each damage mechanism

  📖 [Lemaitre 92], [Cantournet 02], [Angrand 16]

- ➢ Continuous damage evolution for static and fatigue loadings
- ➢ Fatigue formulation depends on maximal equivalent strain



Unloading:

$$\begin{cases} \dot{y}_{max} = 0 \\ \langle\dot{y}\rangle_+ - \dot{y}_{max} = 0 \end{cases}$$

OPERATION PRINCIPLE

- Generalized **cracking kinetics**: $\dot{\bar{\rho}} = f_{stat}(\bar{\rho})g_{stat}(y)\dot{y}_{max} + f_{cycl}(\bar{\rho})g_{cycl}(y)[\langle\dot{y}\rangle_+ - \dot{y}_{max}]$
  - Assumption: linear macroscopic behavior
  - Cyclic loading with constant amplitude

- Analytic resolution by integration

$$\boxed{\bar{\rho}_{stat} = \bar{\rho}_c[1 - \exp\left(-\left\langle\frac{\sqrt{y_{max}} - \sqrt{y^0_{stat}}}{\sqrt{y^c_{stat}}}\right\rangle_+^{p_{stat}}\right)]}$$

N : Number of cycles

$$\boxed{\bar{\rho}_{cycl}(N) = \bar{\rho}_c[1 - [(n-1)(f_{\bar{\rho}_{stat}} + (N-1)I_g)]^{\frac{1}{1-n}}}$$

$$f_{\bar{\rho}_{stat}} = \frac{1}{n-1}\left(1 - \frac{\bar{\rho}_{stat}}{\bar{\rho}_c}\right)^{1-n}$$

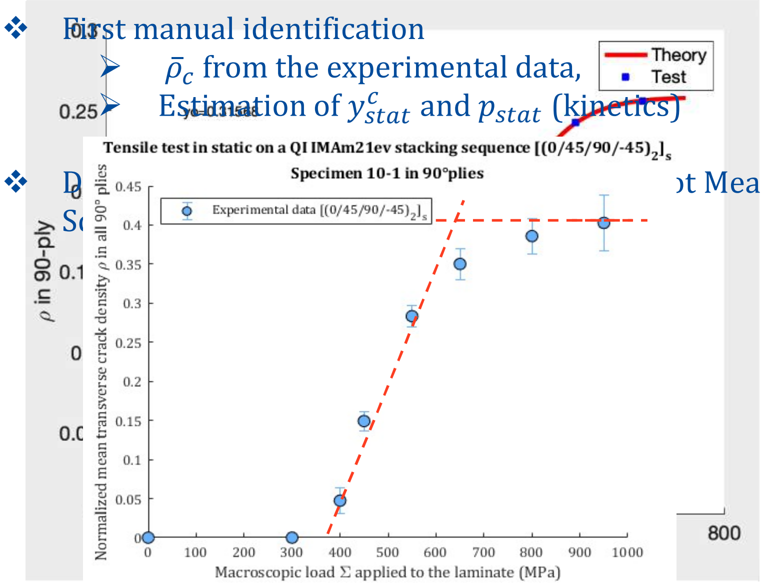
$$I_g = \left\langle\frac{\sqrt{y_{max}} - \sqrt{y^0_{cycl}}}{\sqrt{y^c_{cycl}}}\right\rangle_+^{p_{cycl}} - \left\langle\frac{\sqrt{y_{min}} - \sqrt{y^0_{cycl}}}{\sqrt{y^c_{cycl}}}\right\rangle_+^{p_{cycl}}$$

❖ IDENTIFICATION

❖ Static: 3 parameters to identify

$y^c_{stat}, p_{stat}, \bar{\rho}_c$

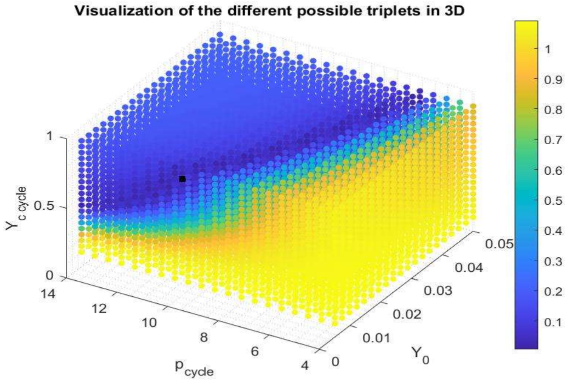
- ❖ First manual identification
  - ➢ $\bar{\rho}_c$ from the experimental data,
  - ➢ Estimation of y^c_{stat} and p_{stat} (kinetics)
- ❖ D ot Mean S



❖ Fatigue: 4 parameters to identify

$y^0_{cycl}, y^c_{cycl}, p_{cycl}, n$

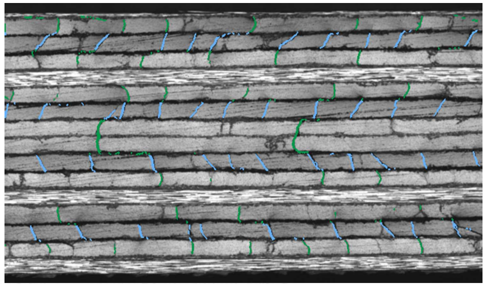
- ❖ Definition of a sweeping range for y^0_{cycl} and p_{cycl},
- ❖ Visualization of RMSE « valleys » and choice of a couple,
- ❖ 3D sweep with y^c_{cycl},
- ❖ Verification with RMSE minimization



❖ RESULTS

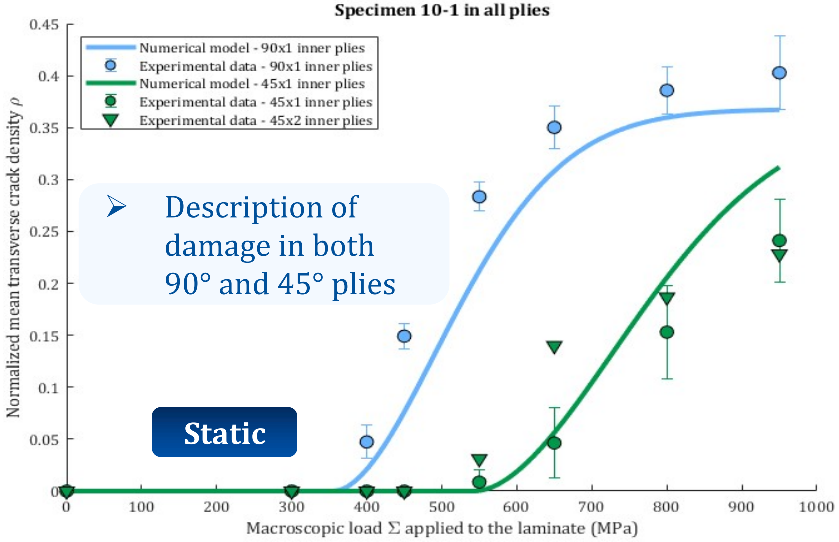
➢ IMA/M21ev

45° plies
90° plies



Tensile test in static on a QI IMAm21ev stacking sequence $[(0/45/90/-45)_2]_s$

Specimen 10-1 in all plies

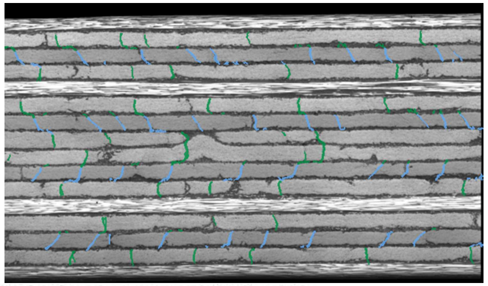


➢ Description of damage in both 90° and 45° plies

Static

10_1
0
45
90
-45
0
45
90
-45
90
45
0
-45
90
45
0

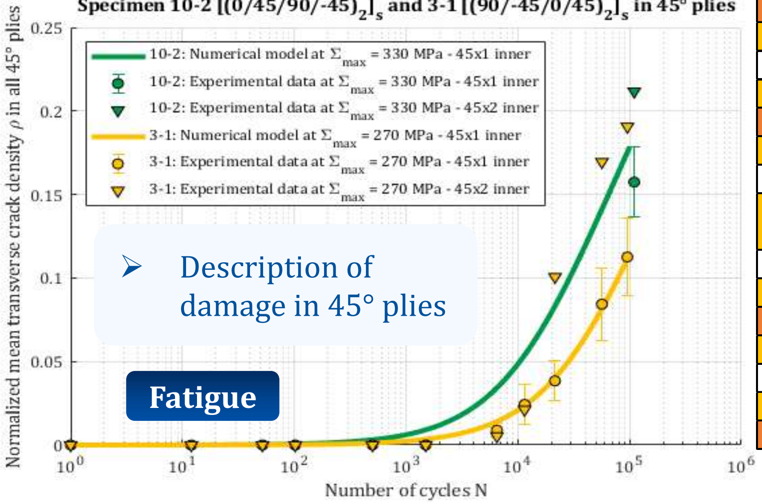
45° plies
90° plies



Fatigue test on two different QI stacking sequences of IMAm21ev

R = 0,05 and f = 5 Hz for different Σ_{max} values

Specimen 10-2 $[(0/45/90/-45)_2]_s$ and 3-1 $[(90/-45/0/45)_2]_s$ in 45° plies



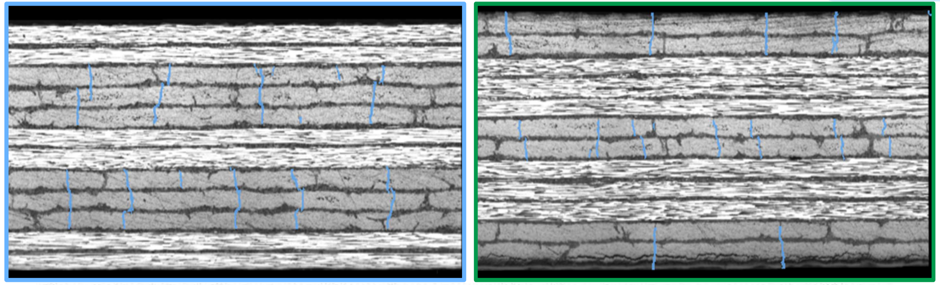
➢ Description of damage in 45° plies

Fatigue

3_1
90
-45
0
45
90
-45
0
45
0
-45
90
45
0
-45
90

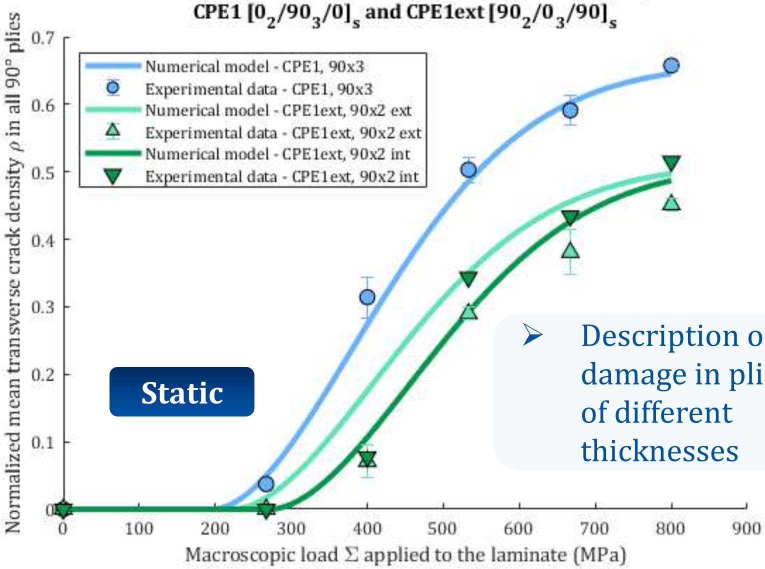
❖ RESULTS

➢ IMA/M21ev



Tensile test in static on two different cross-ply IMAm21ev stacking sequences in 90° plies CPE1 $[0_2/90_3/0]_s$ and CPE1ext $[90_2/0_3/90]_s$

CPE1
0
90
0
90
0



➢ Description of damage in plies of different thicknesses

CPE1ext
90
0
90
0
90

❖ RESULTS

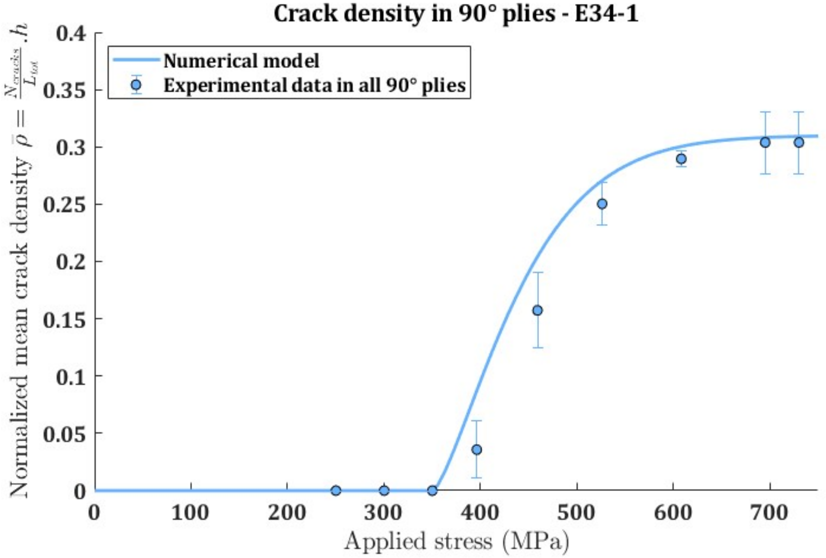
➢ T700GC/M21

Static

Fatigue

Monotonic tensile test for a QI $[45/90/\text{-}45/0/45/90/\text{-}45/0]_s$ stacking sequence

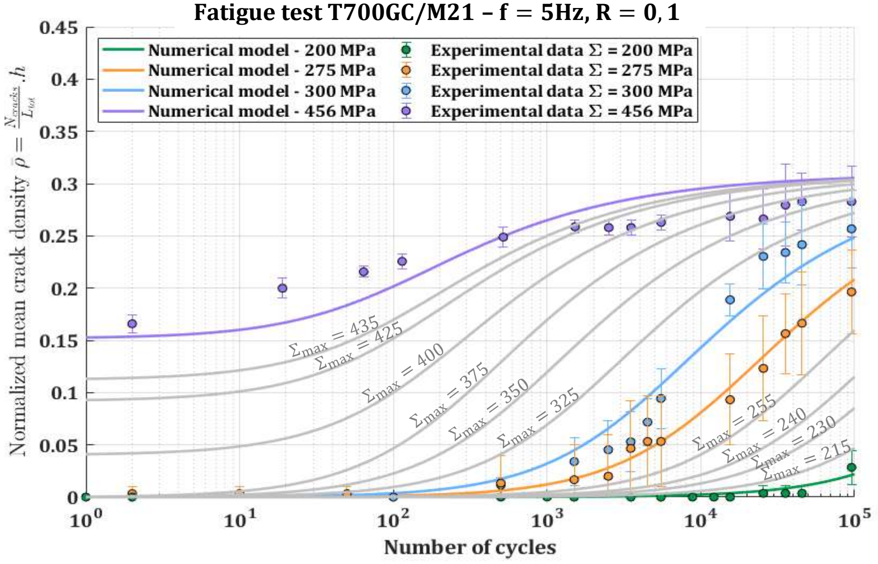
Crack density in 90° plies - E34-1



Normalized mean crack density in 90 plies

QI $[45/90/\text{-}45/0/45/90/\text{-}45/0]_s$

Fatigue test T700GC/M21 – $f = 5\text{Hz}$, $R = 0,1$



➢ Description of damage for different Σ_{max} (MPa) values

CONCLUSION

Originality

- Description of both static and fatigue behaviors,
- Observable variable writing (crack density): direct link between damage and its effects,
- Incremental nature: representative of complex loadings,

Strong hypothesis

- In-plane stress,
- Damage effect on the material's behavior not taken into account yet,
- Non Linear behavior (viscosity & NL elasticity) not taken into account yet

Core strengths

- Highly efficient to describe constant amplitude loadings in fatigue,
- Residual thermal stresses & ply thickness effect taken into account,
- Accessible identification process from constant amplitude fatigue test data,
- Low computational cost & quick running time in the simplified version

❖ PERSPECTIVES

- Improvement of the incremental damage model
  - Effect of damage on the material’s behavior and non linearity,
  - Cumulative damage effect and mean stress effect,
  - Cycle jump method *[Sally 2020]*,
  - Addition of a failure criterion and Finite Element implementation

- Complete experimental campaign on the IMA/M21ev
  - Specific validation tests in fatigue,
  - 10 different stacking sequences, 100 tests planned

Plates	Stacking sequences	Plates	Stacking sequences
Cross-ply laminates (CP1-1 & CP2-1)	$[0_2/90/0_2/90/0/90]_s$ $[90_2/0/90_2/0/90/0]_s$	**[0/±45] laminates (FP1-1 & FP2-1)**	$[45_2/0/-45/0/45/0/-45]_s$
	$[90/0_2/90_2/0_3]_s$ $[0/90_2/0_2/90_3]_s$		$[0/-45_2/45/0/45/0/45]_s$
QI laminates (QI-1)	$[0/45/90/45/0/-45_2/90]_s$	**Double-double (DD1-1)**	$[60/20/-20/-60_2/-20/20/60]_s$
Oriented laminates (O1-1)	$[0/90/45_2/90_2/-45/0/-45/90_2]_s$		$[30/70/-70/-30_2/-70/70/30]_s$

THANK YOU FOR YOUR ATTENTION

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