Efficient uncertainty propagation of a composite single-lap joint and of a stiffened panel

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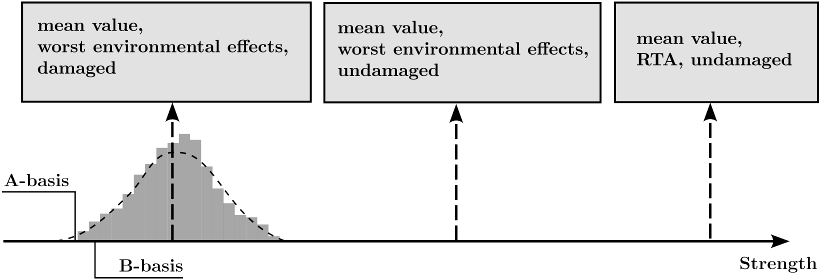
- **Motivation**
- **Parameter sensitivity in non-linear mechanics**
- **Application: composite-aluminum single-lap joint**
- **Application: single stringer compression specimen**
- **Conclusions and prospects**

Motivation

Motivation

> *"The strength value used in a design must be such that if the 'worst of all situations' is combined in service, the resulting structure will still meet the load requirements without failure "*
>
> Kassapoglou C., Design and Analysis of Composite Structures (2013)

- Variability of loads and usage.
- Material scatter.
- Environmental effects.
- Effects of damage.



Parameter sensitivity in non-linear mechanics

Parameter sensitivity in non-linear mechanics

$$\begin{cases} \boldsymbol{K}(h) = \boldsymbol{K}(h^0) + \dfrac{\partial \boldsymbol{K}}{\partial h_i}\Delta h_i + \dfrac{1}{2}\dfrac{\partial^2 \boldsymbol{K}}{\partial h_i \partial h_j}\Delta h_i \Delta h_j + \cdots \\ \boldsymbol{f}^{ext}(h) = \boldsymbol{f}^{ext}(h^0) + \dfrac{\partial \boldsymbol{f}^{ext}}{\partial h_i}\Delta h_i + \dfrac{1}{2}\dfrac{\partial^2 \boldsymbol{f}^{ext}}{\partial h_i \partial h_j}\Delta h_i \Delta h_j + \cdots \\ \boldsymbol{u}(h) = \boldsymbol{u}(h^0) + \dfrac{\partial \boldsymbol{u}}{\partial h_i}\Delta h_i + \dfrac{1}{2}\dfrac{\partial^2 \boldsymbol{u}}{\partial h_i \partial h_j}\Delta h_i \Delta h_j + \cdots \end{cases}$$

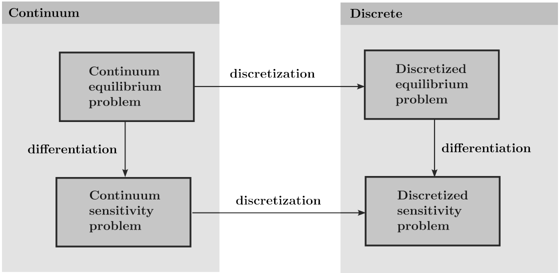


Figure: *Linearization of a generic system through two different approaches.*

Finite Differences (FD)

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}h} \approx \frac{\boldsymbol{u}(h+\Delta h) - \boldsymbol{u}(h)}{\Delta h}$$

- Expensive, accuracy problems.

Adjoint Var. Method (AVM)

$$\boldsymbol{K}(h)\boldsymbol{\lambda}(h) = \left(\frac{\partial \mathcal{G}}{\partial \boldsymbol{u}}\right)^{\top}$$

$$\frac{\mathrm{d}\mathcal{G}}{\mathrm{d}h} = \frac{\partial \mathcal{G}}{\partial h} + \boldsymbol{\lambda}^{\top}\frac{\partial}{\partial h}\left[\boldsymbol{f}_{ext}(h) - \boldsymbol{K}(h)\boldsymbol{u}\right]$$

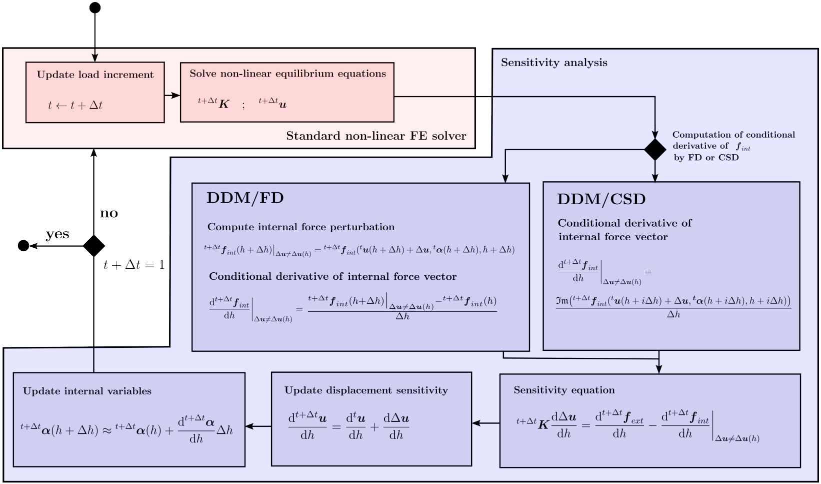
- Scales with the number of functionals $\mathcal{G}$.

Direct Diff. Method (DDM)

$$\boldsymbol{K}\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}h} = \frac{\partial \boldsymbol{f}_{ext}}{\partial h} - \frac{\partial \boldsymbol{K}}{\partial h}\boldsymbol{u}$$

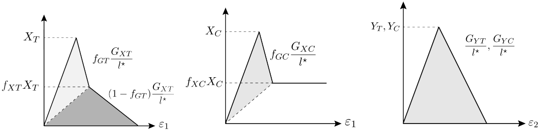
- Scales with the number of design variables h.

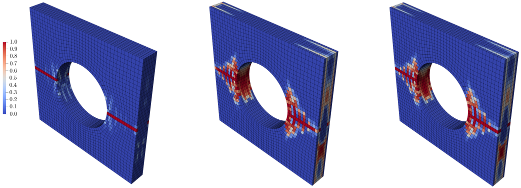
Parameter sensitivity in non-linear mechanics

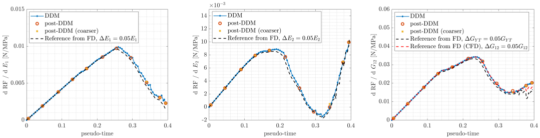


- An **implicit** solution scheme is required.
- Valid for both statics and dynamics.
- Added computational cost (semi-analytical):
  - 1 linear system
  - 2 evaluations of $\boldsymbol{f}_{int}$

Parameter sensitivity in non-linear mechanics







- Applications to composite damage mechanics.

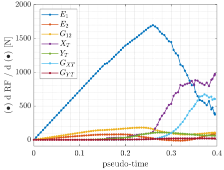


Figure: *Sensitivities along simulation time.*

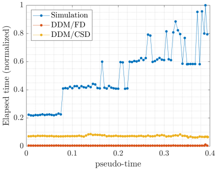
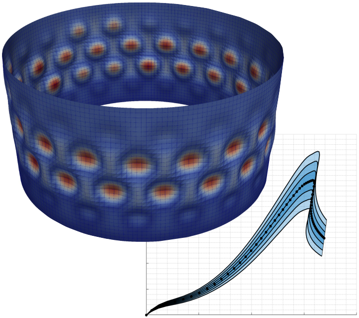


Figure: *Simulation time vs. sensitivity time.*

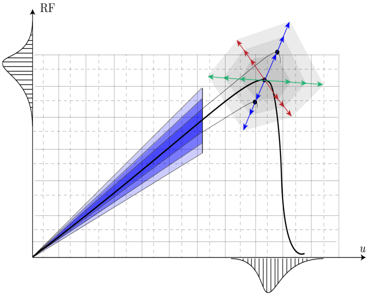
Parameter sensitivity in non-linear mechanics

How to deal with critical points where ${}^{t+\Delta t}K$ is singular ?

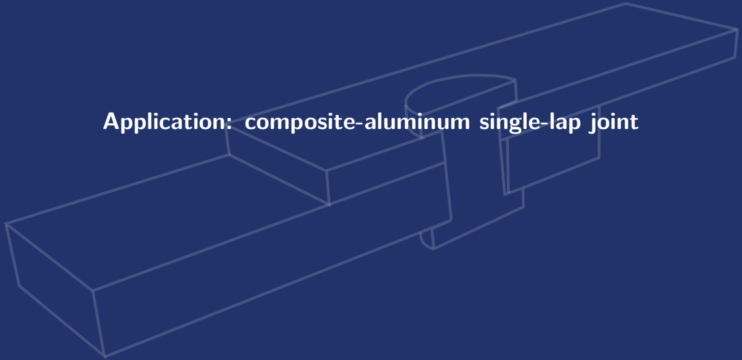
Arc-length solution schemes



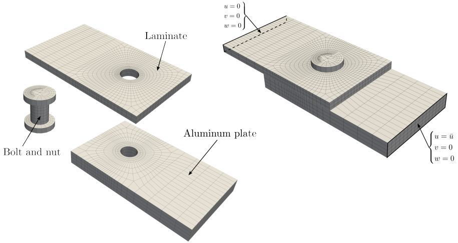
Restart the simulation for the linearized system



Application: composite-aluminum single-lap joint



Composite-aluminum single-lap joint



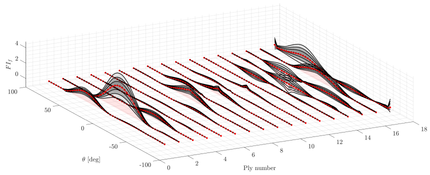
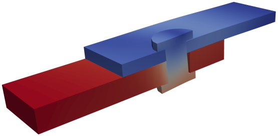
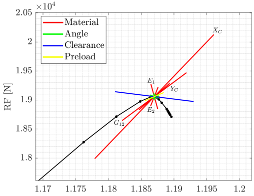


Figure: *failure index (FI) along laminate thickness.*





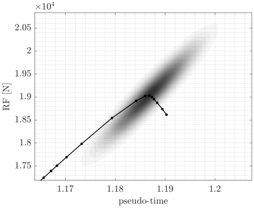
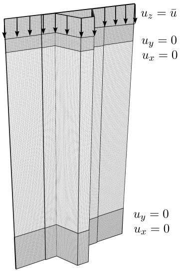
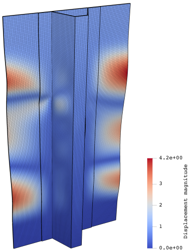
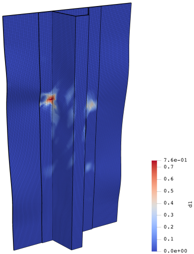


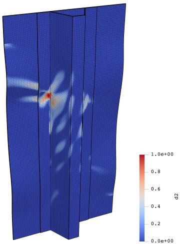
Figure: *Maximum load spread.*

Application: single stringer compression specimen









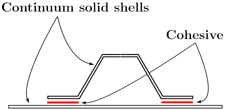
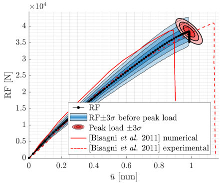
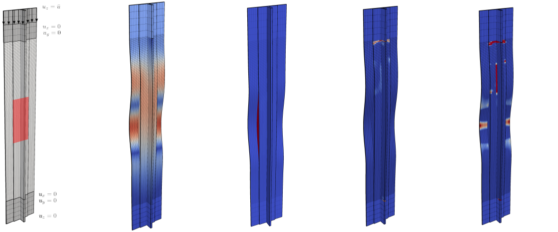


Figure: *Cross-section.*

Total length	300 mm
Free length	240 mm
Width	150 mm
Skin layup	$[45/90/\text{-}45/0]_s$
Stiffener layup	$[\text{-}45/0/45/\bar{0}]_s$
Ply thickness	0.125 mm
Stiffener height	30 mm
Stiffener width	73 mm



Single stringer compression specimen (2)



Total length	400 mm
Free length	300 mm
Width	64 mm
Skin layup	$[90/\pm45/0]_s$
Stiffener layup	$[(\pm45)_3/0_6]_s$
Ply thickness	0.125 mm
Stiffener height	14 mm
Stiffener width	32 mm
Debond length	80 mm

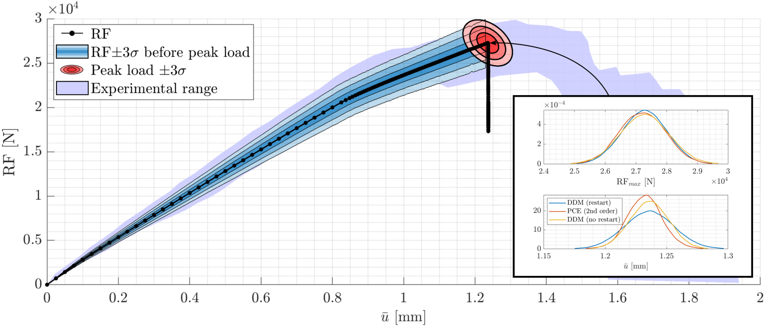


Figure: *Variability of load-displacement curve.*

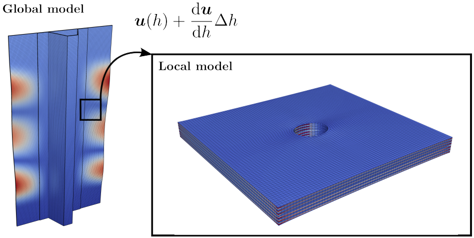
Conclusions and prospects

Conclusions and prospects

- Focus on parameter sensitivity for non-linear mechanics using a direct differentiation approach.
- Application of first-order sensitivity information to approximate the variability of the mechanical response of composite structures:
  - large displacements.
  - intra-laminar damage.
  - inter-laminar damage.
  - contact

Prospects

- Global/local study with damage in local model.
- Iterative solution between global and local models.



Thank you for the attention